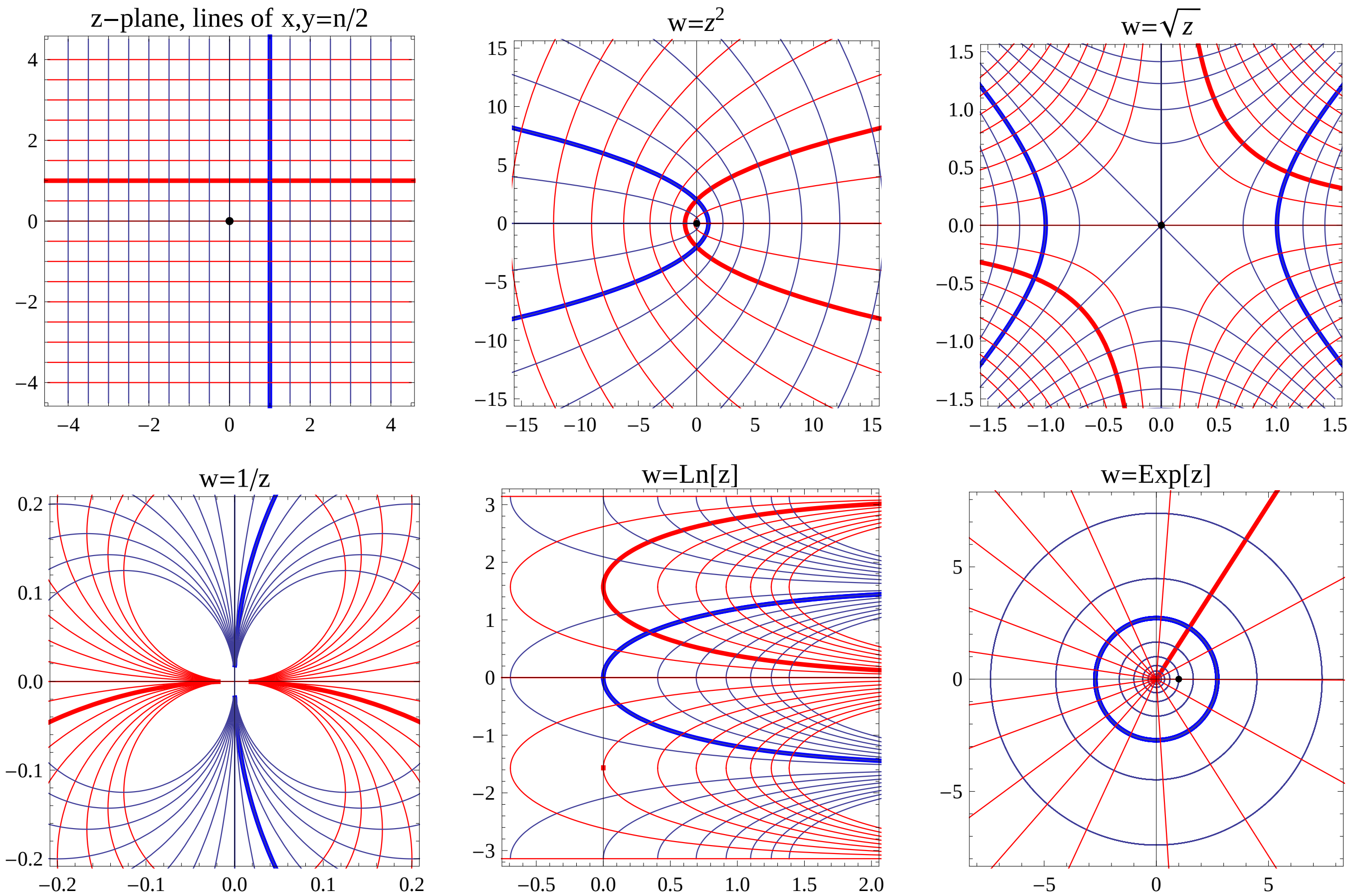
PHYS20672 Complex Variables and Vector Spaces: Handout 1



The first picture (top left) shows lines of constant x (blue) and y (red) in the complex z-plane.

The remaining five pictures show where the lines of constant x or y map to in the complex w-plane for a variety of mappings $w = f(z)$. Lines have been drawn for integer and half-integer values of x and y. The lines $x = 1$ and $y = 1$ are thicker than the others, and the point $w = f(0)$ is shown as a black dot.

For $w = z^2$, the two lines $x = \pm a$ map to a single curve in the w-plane; the same is true for $y = \pm b$. For $w = \sqrt{z}$, each of the grid lines in the z-plane maps to two curves in the w-plane, because the square-root function is doubly valued.

For $w = 1/z$ the blank circular spaces would fill in if we included lines at higher and higher values of x and y. For this and for $w = e^z$, the point $w = 0$ does not correspond to finite z (it is not in the range of the function).

For $w = e^z$, lines of constant x map to the circles $|w| = e^x$; lines of constant y map to radial lines of constant $\arg w = y$. If you find the left half of this plot as confusing as I did, you should note that lines of constant y have been included up to $|y| = 4$, which is greater than π.

For $w = \operatorname{Ln} z = \ln|z| + i\operatorname{Arg} z$, $v = \operatorname{Arg} z$ is restricted to the range $-\pi$ to π. If we had plotted the multiply-valued function $w = \ln z = \operatorname{Ln} z + 2\pi ik$, the pattern shown would repeat, vertically shifted by $2\pi k$, to cover the whole plane. Note that for this plot the scales of the two axes are not the same, which is why the lines do not seem to cross at right-angles.