Illustration of the argument theorem

We illustrate this for a cubic polynomial:

In[157]:=
```
f[z_] := 2 z + z^3
```

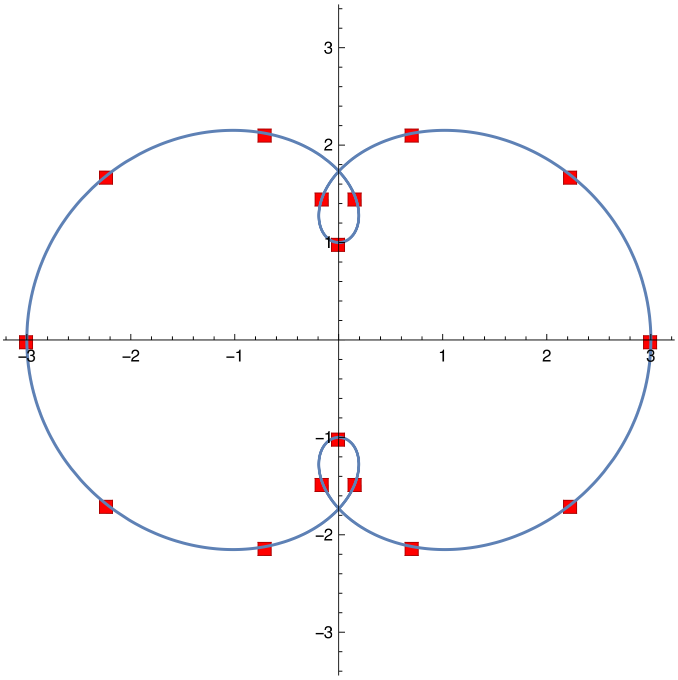
First set up a table of `w=f(z)` evaluated at 16 points on the unit circle in the `z`-plane:

In[182]:=
```
w = Table[{Re[f[Exp[I θ]]], Im[f[Exp[I θ]]]} /. θ → 2.0 Pi m / 16, {m, 16}];
```

Plot these and, since it is easy to do, we fill in the gaps:

In[183]:=
```
Show[ListPlot[w, PlotMarkers → {Red, Small}],
 ParametricPlot[{Re[f[Exp[I θ]]], Im[f[Exp[I θ]]]}, {θ, 0, 2 Pi}],
 PlotRange → {{-3.1, 3.1}, {-3.1, 3.1}}, AspectRatio → 1]
```

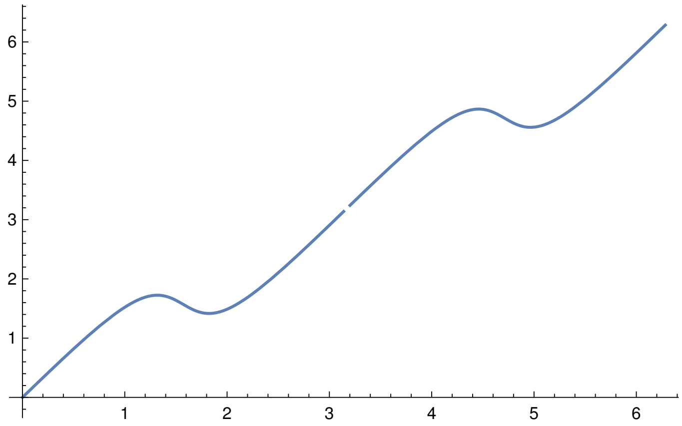
Out[183]=



Note that the curve (in the `w`-plane) encircles `w=0` once. There is therefore only one solution of `f(z)=0` with `|z|<1`. It is the solution `z=0`.

```
In[186]:= Block[{z = Exp[I θ]},
  Plot[Arg[f[z]] + If[θ < Pi, 0, 2 Pi], {θ, 0, 2 Pi}]]
```

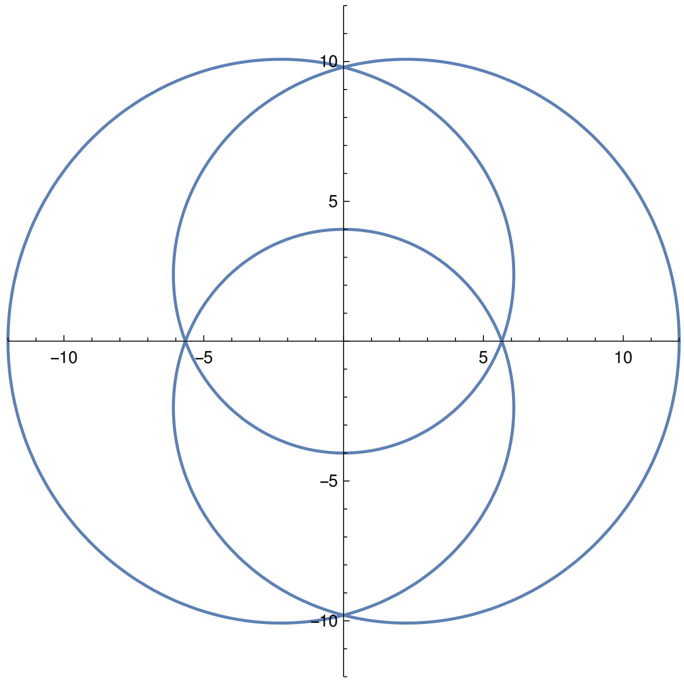
Out[186]=



Repeating the first plot with `|z|=2` gives the following plot in the `w`-plane:

```
In[180]:= ParametricPlot[{Re[f[2 Exp[I θ]]], Im[f[2 Exp[I θ]]]},
  {θ, 0, 2 Pi}, PlotRange → {{-12, 12}, {-12, 12}}, AspectRatio → 1]
```

Out[180]=



The curve (in the `w`-plane) encircles `w=0` three times. There are therefore three solutions of `f(z)=0` with `|z|<2`. They are `z=0` and the two square roots of `-2`.