PC4902: Elements of QMBT, Pt 2 Problems 2

Key: **E**asy; **M**oderate; **D**ifficult

1. [EM] In the Thomas–Fermi approximation, the screened electrostatic potential $\phi(\mathbf{r})$ near a charge Ze in a free-electron metal with electron density n satisfies

$$\nabla^2\phi - q_0^2\phi = -Ze\delta(\mathbf{r})/\epsilon_0, \tag{1}$$

where $q_0^2 = 3ne^2/(2\epsilon_0 E_F)$ is the square of the Thomas–Fermi wave number. Show that $\phi(\mathbf{r}) = Ce^{-q_0 r}/r$ is a solution of (1) for $r \neq 0$. Explain, using a physical argument, why the constant C should be $Ze/4\pi\epsilon_0$.

Evaluate q_0 and k_F for aluminium, regarding it as a trivalent free-electron metal with face-centred cubic lattice parameter $a = 4.05$ Å. [There are *four* atoms in the conventional unit cell of the face-centred cubic structure.]

Note: The radial part of $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{2}{r}\dfrac{\partial}{\partial r}$.

[Ans: $q_0 = 2.05$ Å^{-1}; $k_F = 1.75$ Å^{-1}]

2. [E] Electrons of energy 20 keV are passed through a 1000-Å film of aluminium giving the energy-loss spectrum shown below. Explain the origin of the large peaks. Find the electron density in aluminium and hence estimate the lattice parameter.

[Diagram not reproduced here. Energy-loss spectrum shows peaks at 0, 15 and 30 eV.]

[Ans: $a \simeq 4.2$ Å]