PC4902: Elements of QMBT, Pt 2 Coursework 2

Please return your work to the Undergraduate Office on or before **Friday 12 May**.

(a) The eigenvectors of a Hermitian matrix form a complete set of vectors which may be taken to be orthonormal. Use this result to show that the largest (smallest) diagonal matrix element of a Hermitian matrix can be no greater than (less than) its largest (smallest) eigenvalue.

(b) Consider a pair of spins of equal magnitude S. By writing $\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$ in terms of $(\hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2)^2$, show that the largest diagonal matrix element that $\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$ can have is S^2.

(c) Show that the smallest diagonal matrix element that $\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$ can have is $-S(S+1)$.

(d) For a set of spins $\{\hat{\mathbf{S}}_i\}$ of equal magnitude S, consider the antiferromagnetic Heisenberg Hamiltonian

$$\hat{H} = \sum_{i<j} J_{ij}\, \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$$

where $J_{ij} = J_{ji} \geq 0$ for all $i \neq j$. Show that the energy eigenvalues E_n lie in the range

$$-S(S+1) \sum_{i<j} J_{ij} \;\leq\; E_n \;\leq\; S^2 \sum_{i<j} J_{ij}.$$