

# Simplified approaches to opinion dynamics modelling

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# Outline

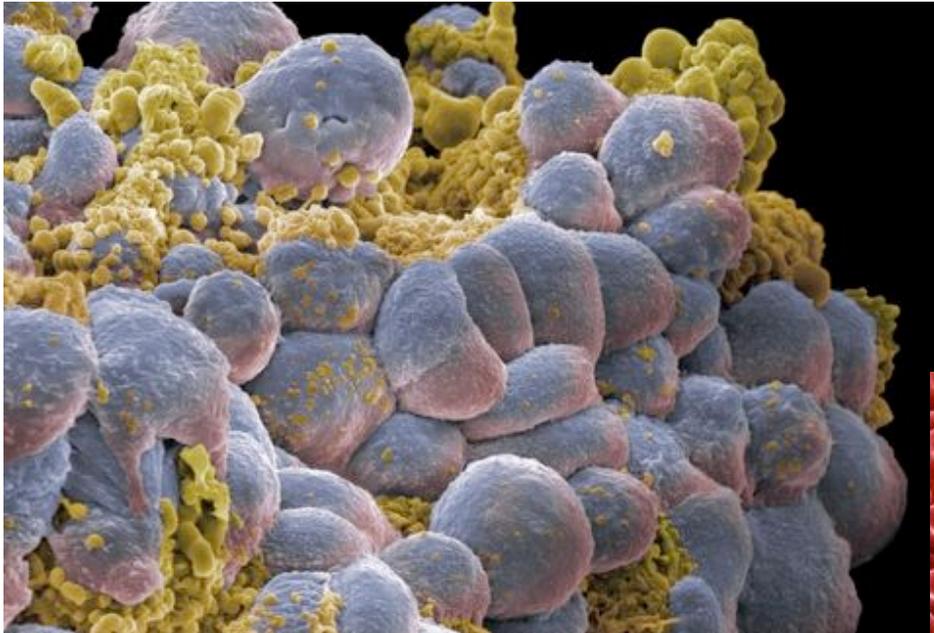
Modeling social systems: population, interactions, dynamics

The case of opinion dynamics: the Deffuant model

Heuristics and decision making: from psychology to a simplified dynamical framework

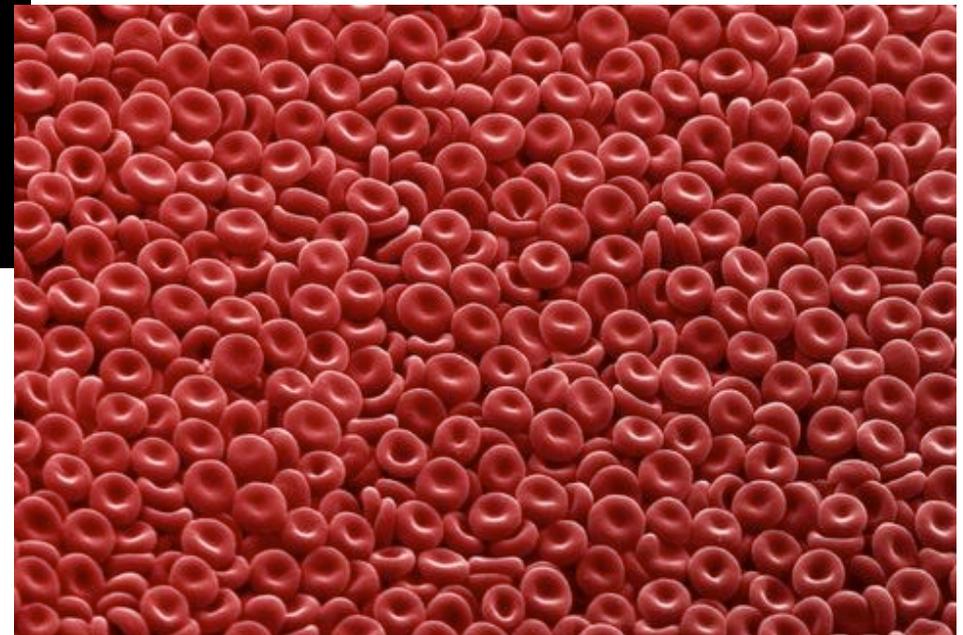
Experiments vs. models: the virtual community laboratory

# Natural phenomena often involve large assembly of simultaneously interacting bodies

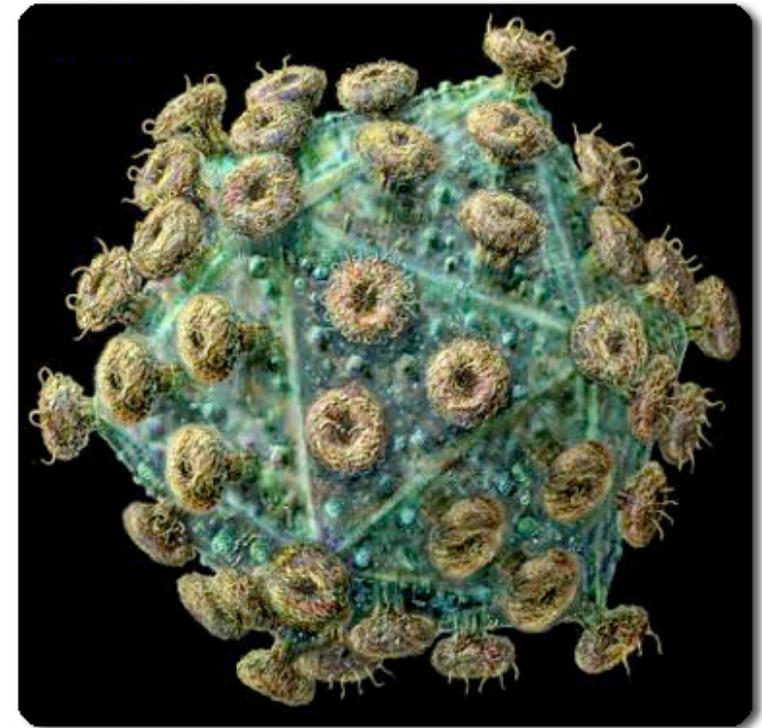
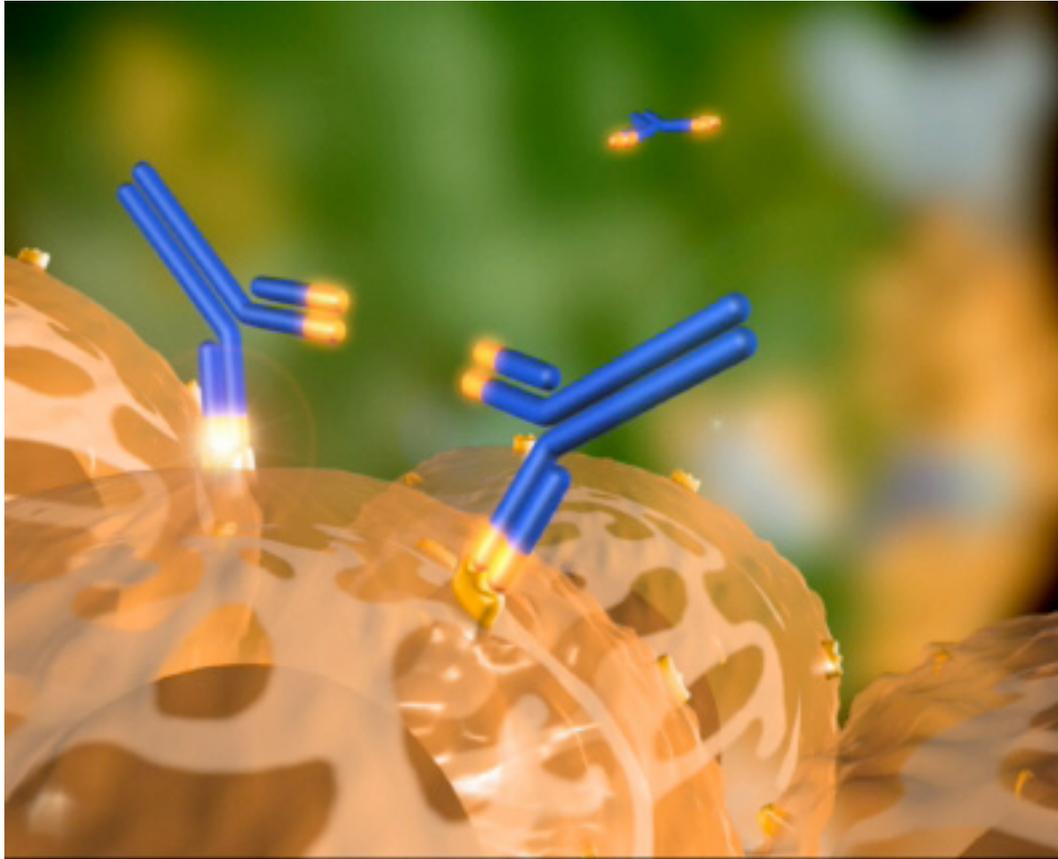


Interacting cells

A red carpet of blood cells

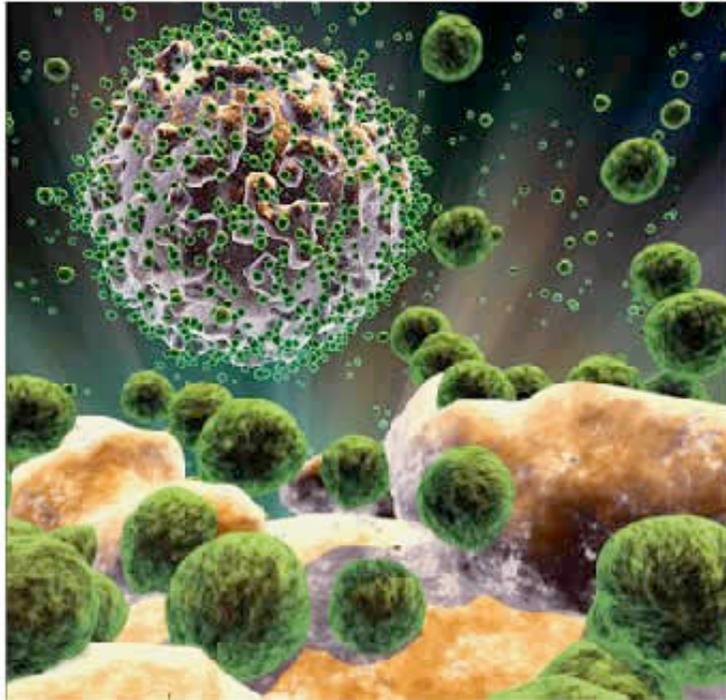


## Different processes occur at different scales...



Different molecular actors belonging to distinct population in mutual competition (e.g. virus and antibodies)

**Modeling** the mutual interaction between interacting species is indeed a **cross-disciplinary** field of investigations



From **life sciences** (e.g. biology, ecology...)

...to e.g. **social systems** (traffic models, crowd dynamics)

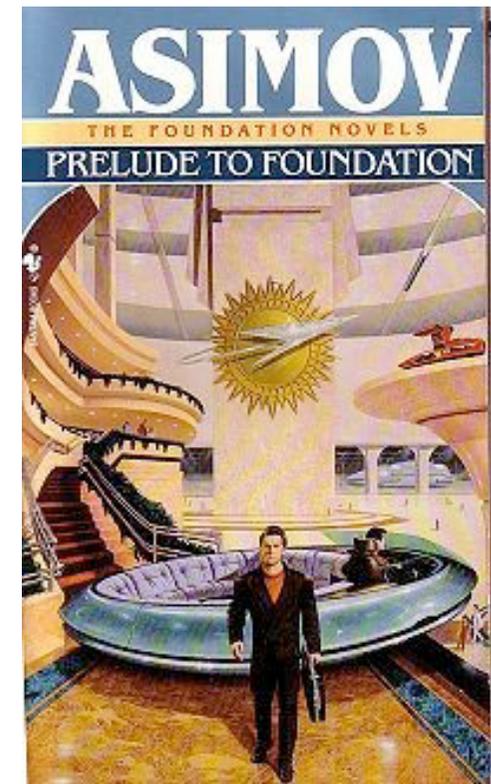


They all **share** similar tools and mathematical techniques

"Your Imperial Majesty ... consider the manner in which **scientists have dealt with subatomic particles**. There are enormous numbers of these, each moving or vibrating in random and unpredictable manner, but this chaos turns out to have an underlying order, so that we can work out a quantum mechanics that answers all the questions we know how to ask.

**In studying society, we place human beings in the place of subatomic particles**, but now there is the added factor of the human mind. Particles move mindlessly; human beings do not. To take into account the various attitudes and impulses of mind adds so much complexity that there lacks time to take care of all of it."

Prelude to Foundation by Isaac Asimov, 1988



It is particularly interesting to study the inherent dynamics of relatively small groups.

Individuals can debate on a specific issue and consequently modify their opinion or beliefs.

The group can spontaneously organize an internal structure, e.g. by identifying the leader.

The opinion forming can be externally influenced by a “propaganda”

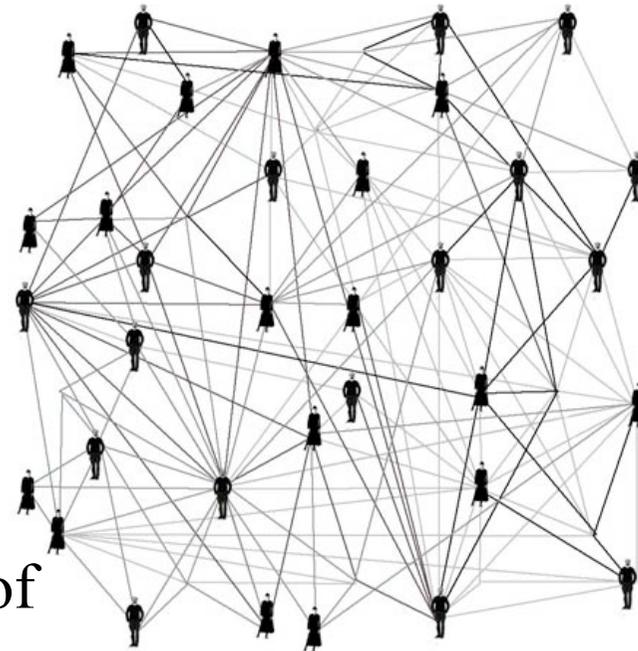
How to model these (and many more) effects within a self-consistent dynamical picture?



## The physicist's approach: 1D opinion representation

Consider  $N$  individual and imagine individual  $i$  to bear a specific (scalar) opinion  $O_i$  on a given topic

Individuals are linked together, via short vs. long range couplings, taking part of



People are nodes of an abstract network of connection, which can possibly account for the explicit notion of space.

## Two different philosophies



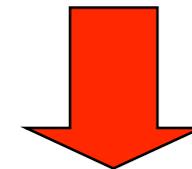
Binary opinion  
(0,1) / up-down

Ising model, Curie-Weiss,  
spin glasses

*Magnetized phases correspond to  
configuration where consensus in the  
population has been reached*



Continuous opinion  
 $0 < O_i < 1$

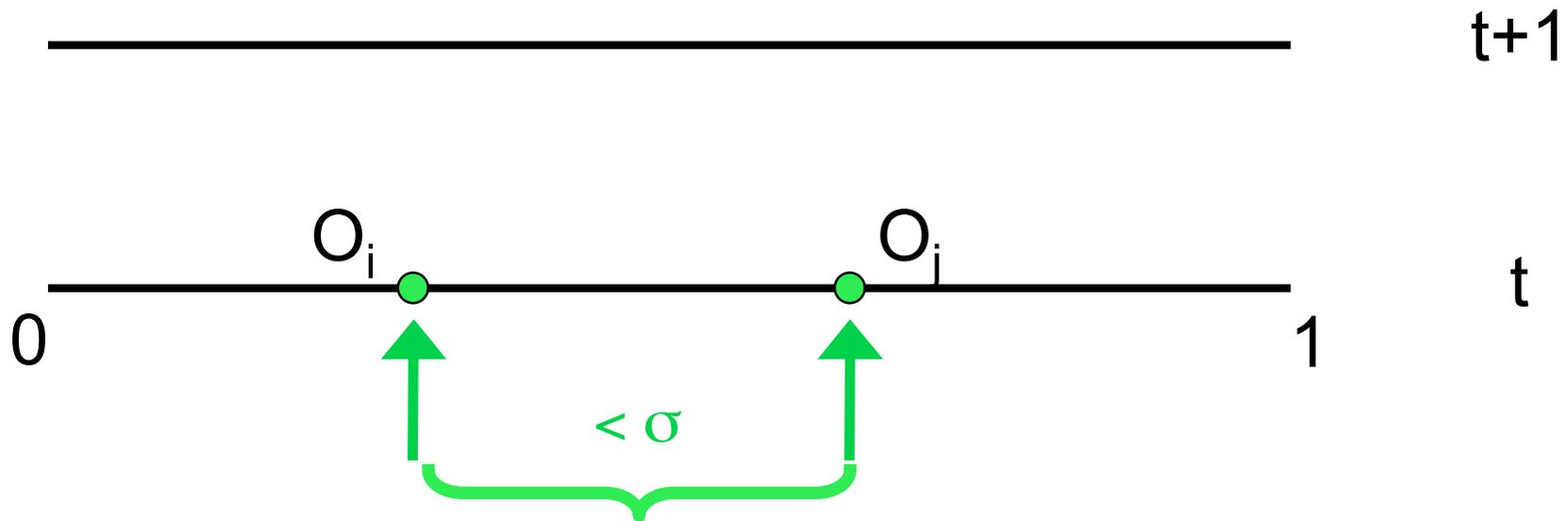


**Deffuant et al.**

Assigned specific rules of  
microscopic interactions

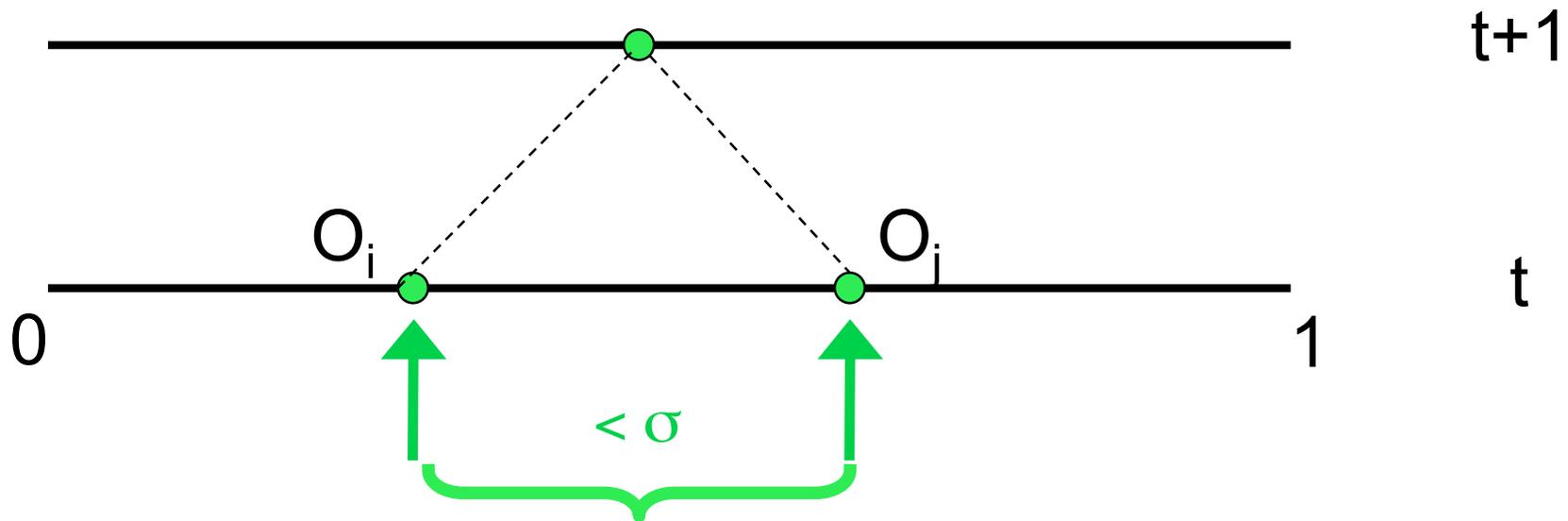
# Deffuant's model

$$\left\{ \begin{array}{ll} |O_i(t) - O_j(t)| < \sigma & \longrightarrow O_i(t+1) = O_i(t) + \mu|O_j(t) - O_i(t)| \\ |O_i(t) - O_j(t)| > \sigma & \longrightarrow O_i(t+1) = O_i(t) \end{array} \right.$$



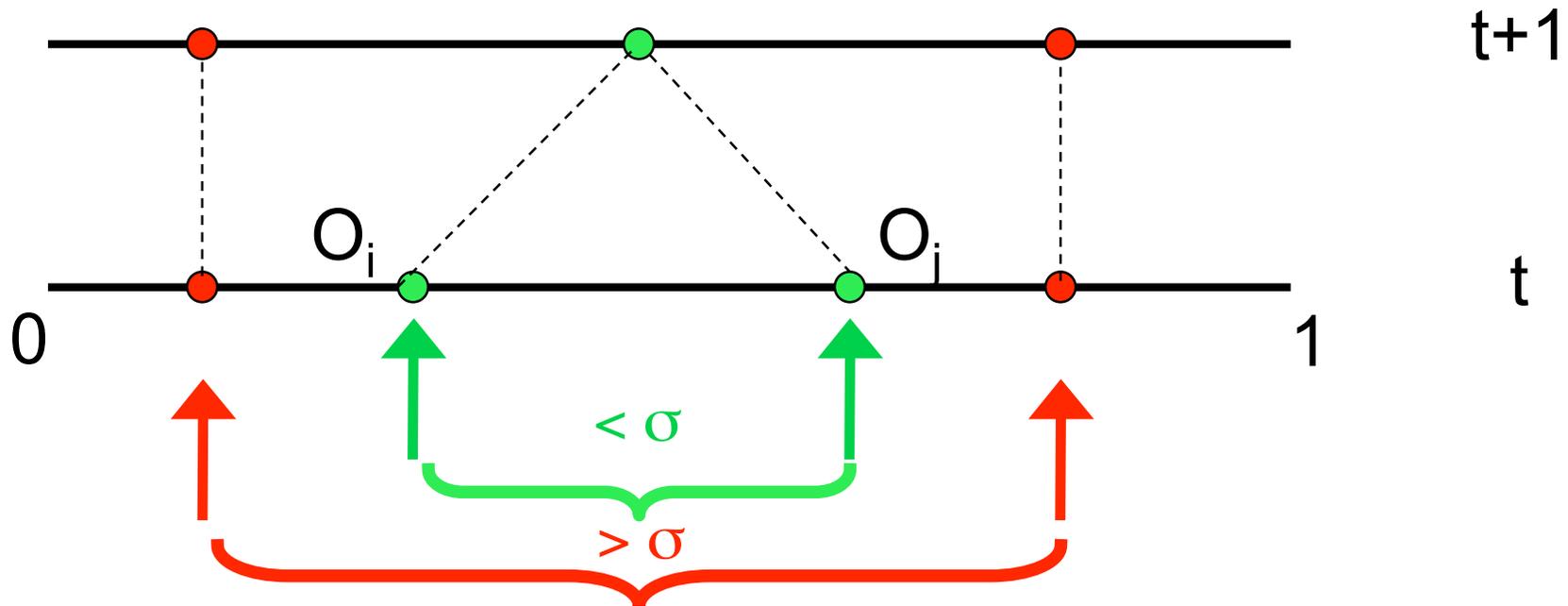
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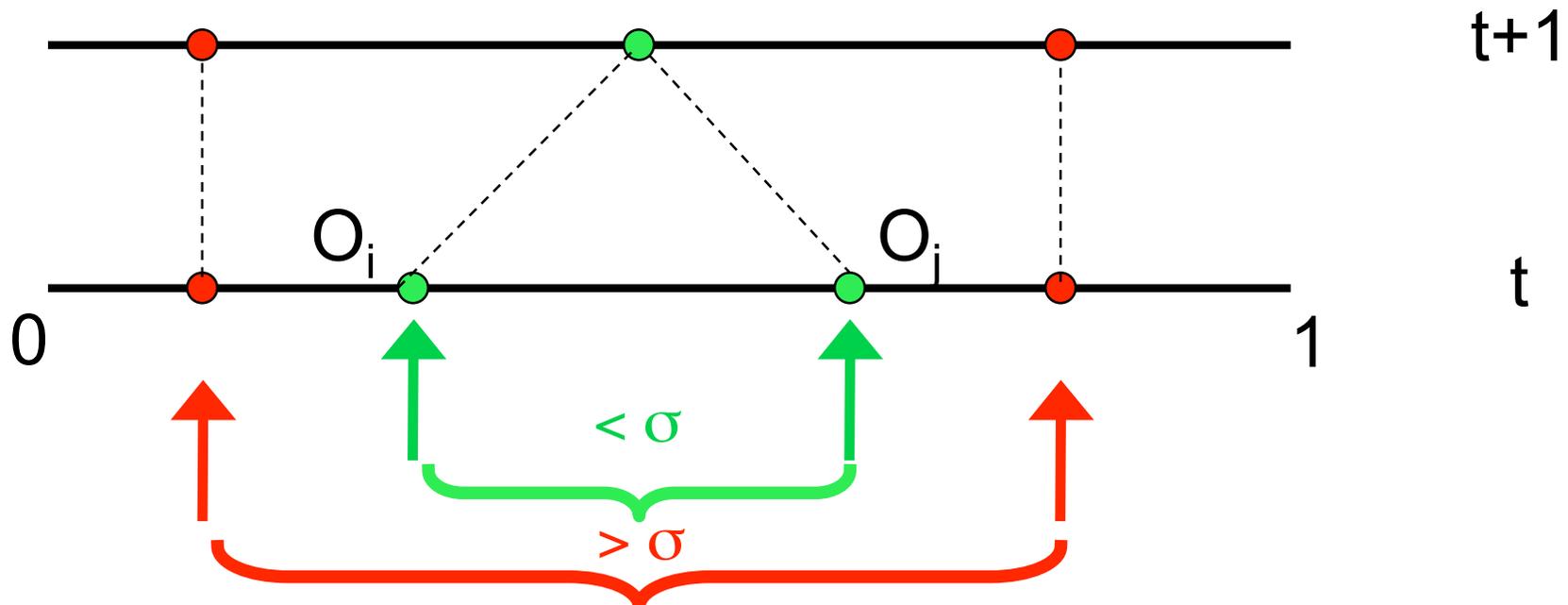
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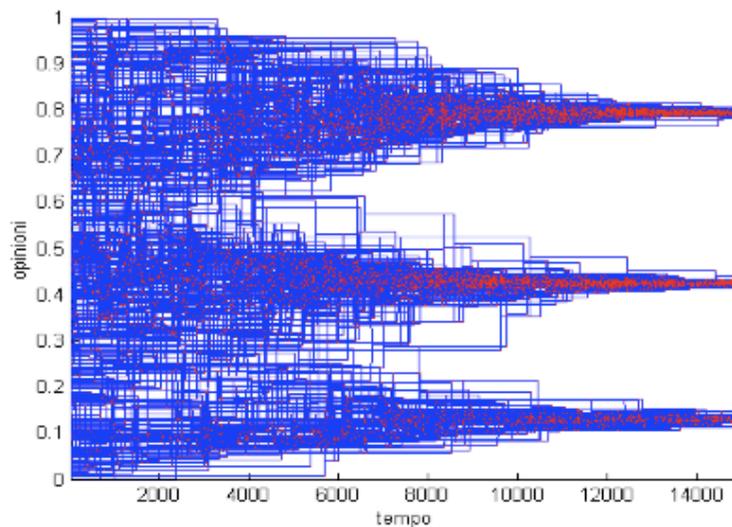
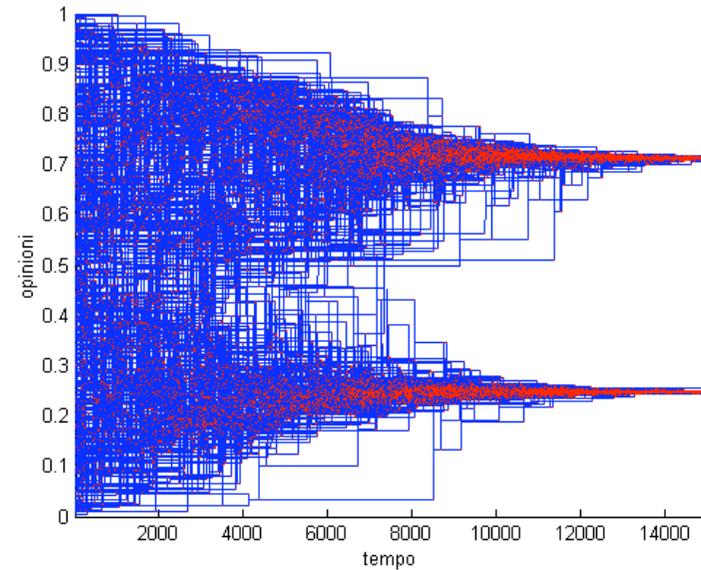
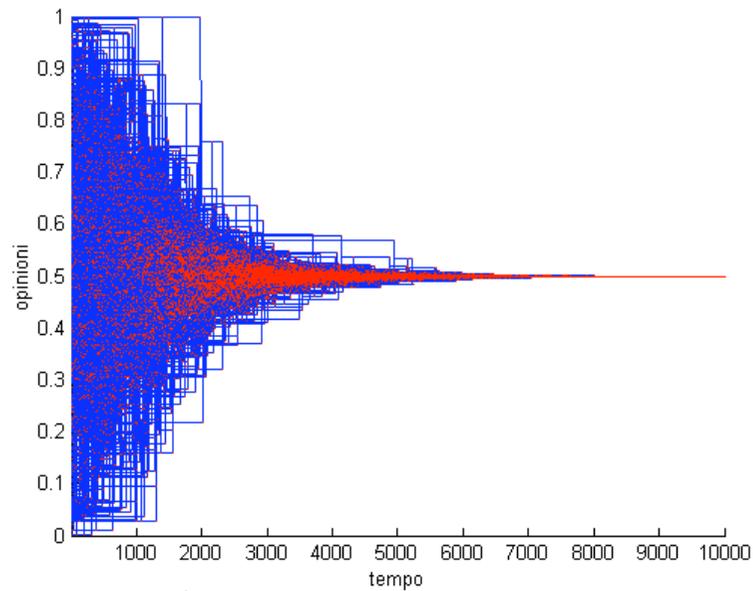


# Deffuant's model

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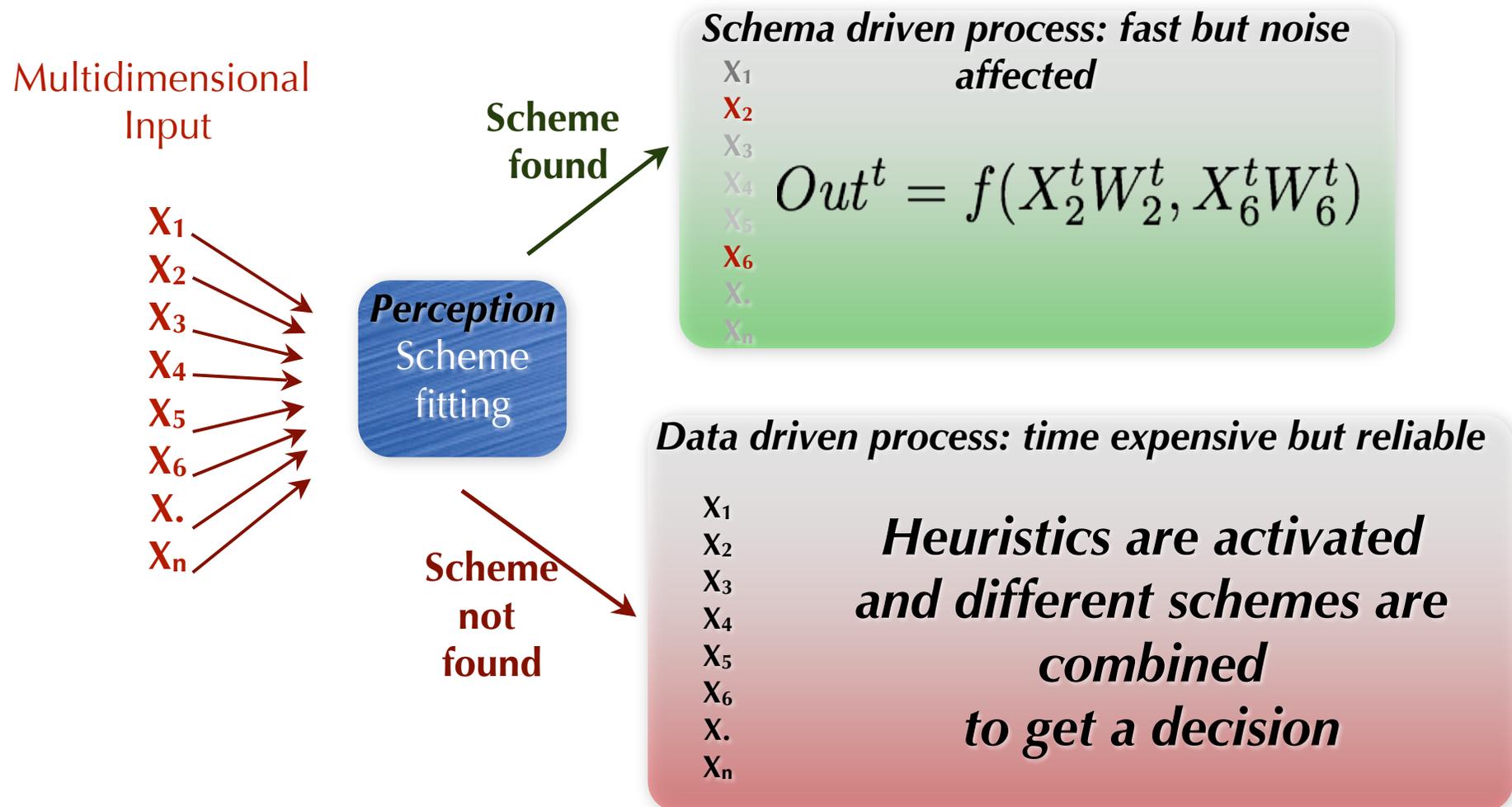


## Opinion vs. time: different threshold amount



The number of clusters  
scales as  $1 / (2 \sigma)$

In reality the process of **opinion formation** is rather complicated and involve detailed **mechanisms** studied by the **psychological literature**.



In general improving the model would necessitate...

Elucidate agents' cognitive processes  
(microscopic rules)

Isolate collective (social) behaviours  
(macroscopic observables)

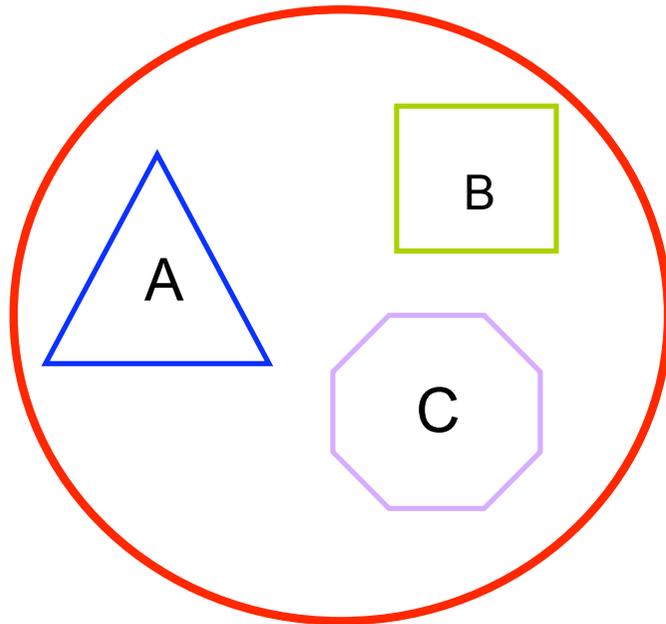


The degree of macroscopic complexity defines  
the level of microscopic coarse graining

# The agent: decision making and cognitive strategies

## Heuristics

In psychology, **simple**, **efficient** rules, hard-coded by evolution to explain decision making, judgment, and problem solving.



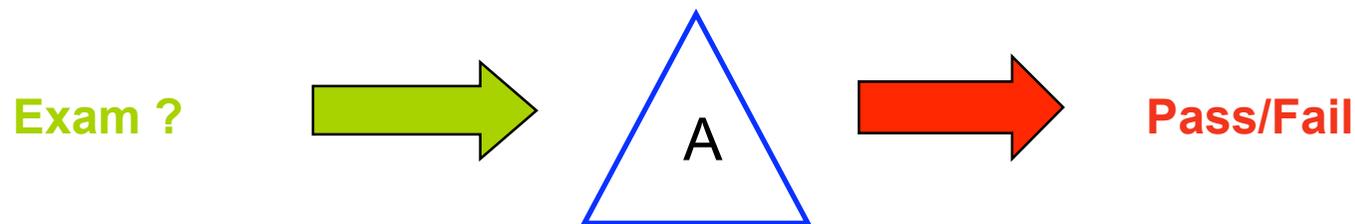
A. Representativeness

B. Availability

C. Anchoring

## A. Representativeness

Evaluate the similarity of selected features with respect to reference templates and judge accordingly.



**Example** Mike is a student you haven't met before. Will he pass the exam? To decide you compare with the *prototype* of successful student and take a decision.

## B. Availability

Assess the probability of an event on the basis of direct experience. The latter refers to the specific environment where the agent acts.

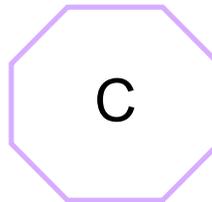


**Example** You live in Sweden and you are asked to estimate the percentage of individual with blond hair all over the world. Based on you daily experience (*feedback from the society*) you go for 70%.

## C. Anchoring

When asked to make a decision under uncertainty conditions, one refers to well established '**anchors**', which are adjusted *ad hoc* to serve the scope.

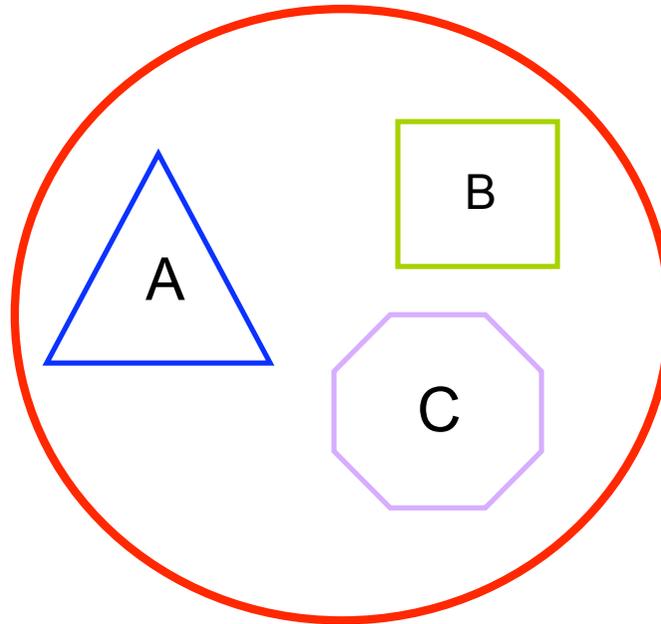
The basket  
team is losing?



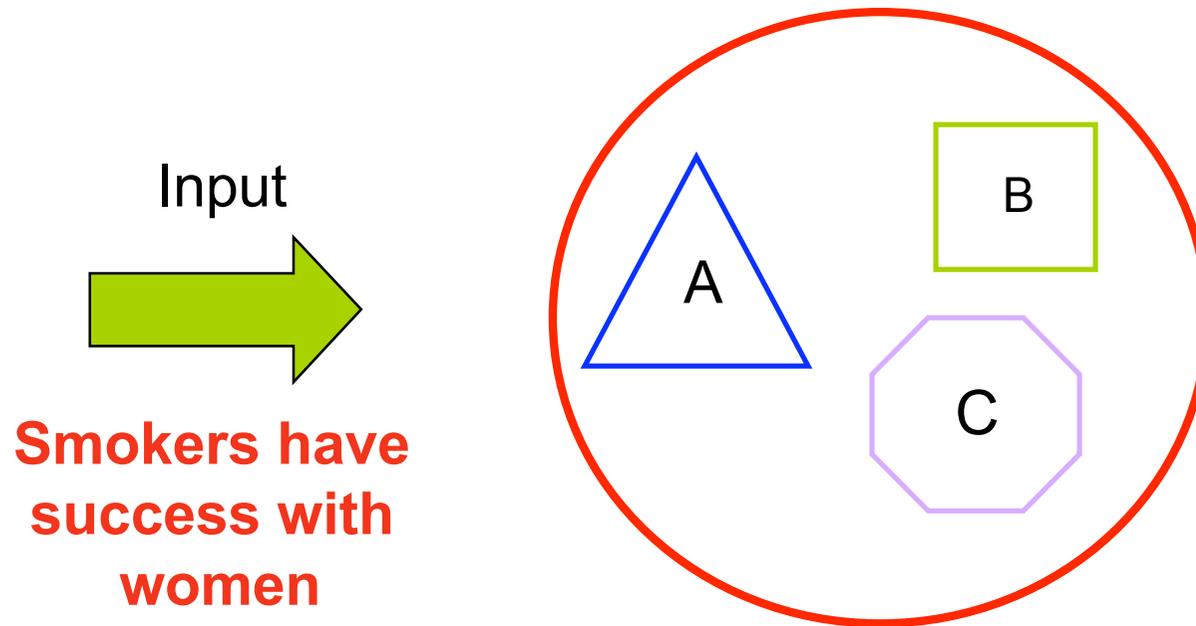
Change a  
player

**Example** I am quite skilled in football and I know all about it. I am watching a basket match on TV. To interpret this new input I make use of an adjusted version of the schemes originally devised for the football.

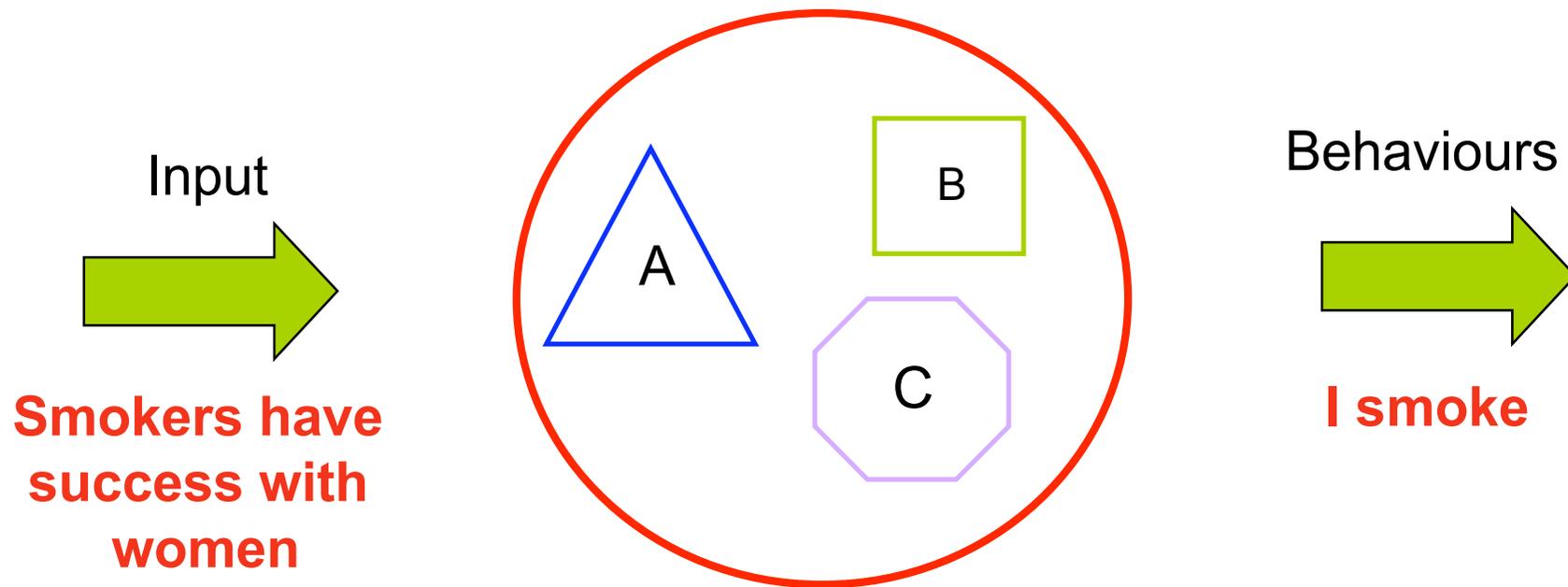
Heuristics and coexistence of multiple  
inputs: the **cognitive dissonance** concept



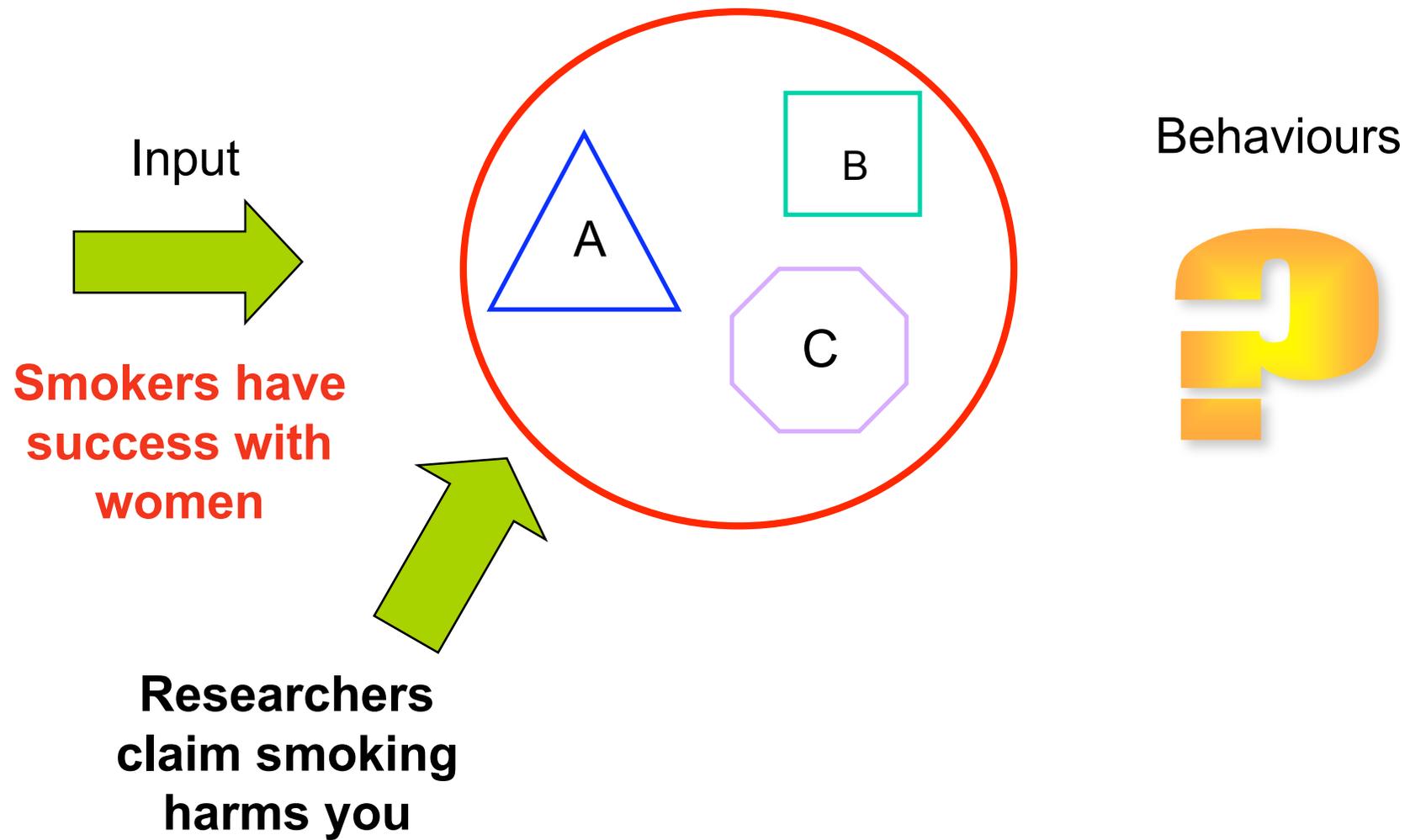
Heuristics and coexistence of multiple inputs: the **cognitive dissonance** concept



Heuristics and coexistence of multiple inputs: the **cognitive dissonance** concept

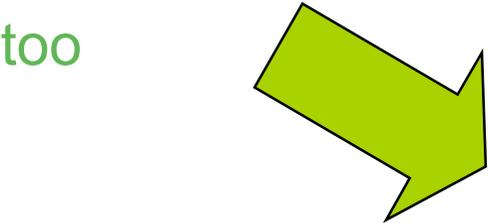


A,B,C vs. cognitive dissonance

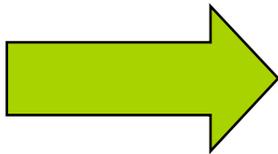


A,B,C vs. cognitive dissonance

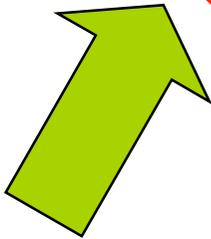
Non smokers  
are succesfull  
too



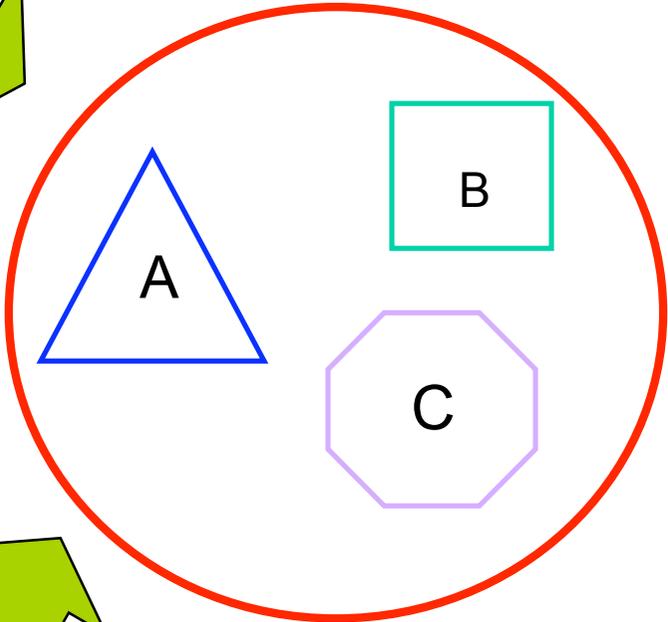
Input



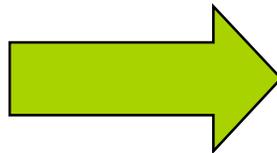
Smokers have  
success with  
women



Researchers  
claim smoking  
harms you



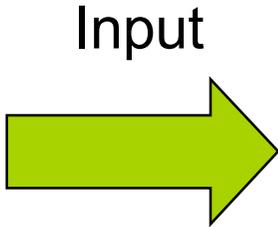
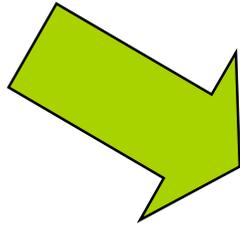
Behaviours



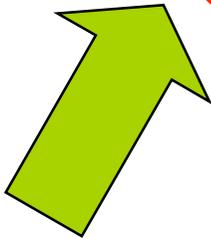
I stop smoking

A,B,C vs. cognitive dissonance

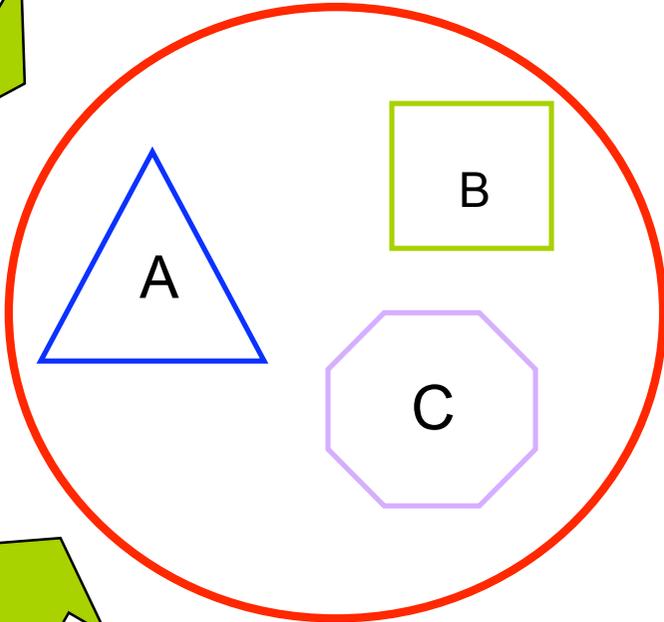
Non smokers are successful too



Smokers have success with women



Researchers claim smoking harms you



Behaviours



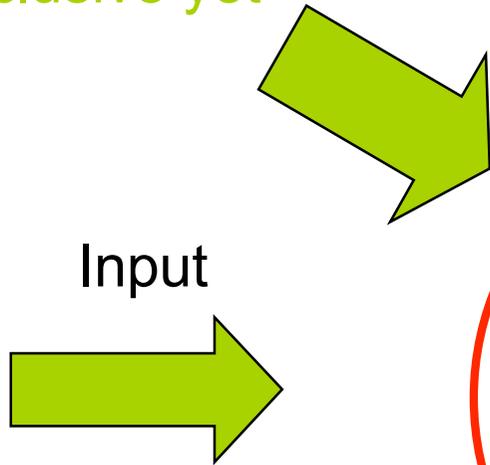
I stop smoking

ACTION I

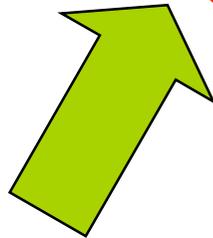
Researchers are a *trustable source* and I seek for further inputs that help mitigating the unpleasant tension state

A,B,C vs. cognitive dissonance

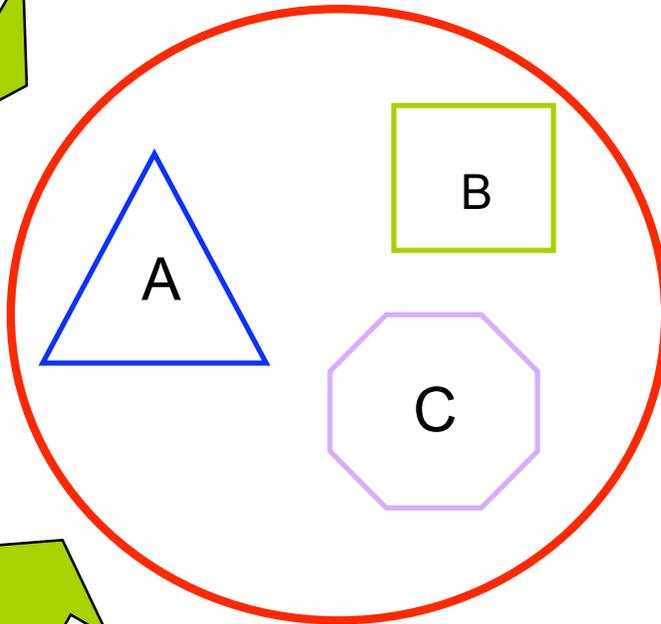
Research is not conclusive yet



Smokers have success with women



Researchers claim smoking harms you



Behaviours



I keep on smoking

ACTION II

*Research is still ongoing (researchers aren't a trustable source) and I keep on smoking*

Developing a **comprehensive model** which accounts for the aforementioned mechanism is a **challenging task**.

The number of **variables/parameters is extremely large**: when trying to match the **(few) experimental observables**, one probably faces a **ill defined problem**.

**Analytical progress is difficult** and one is forced to rely on **simulations**.

*Can one incorporate some of the above mentioned ingredients in a simplified dynamical scheme, which extends over the simplified Deffuant's model?*

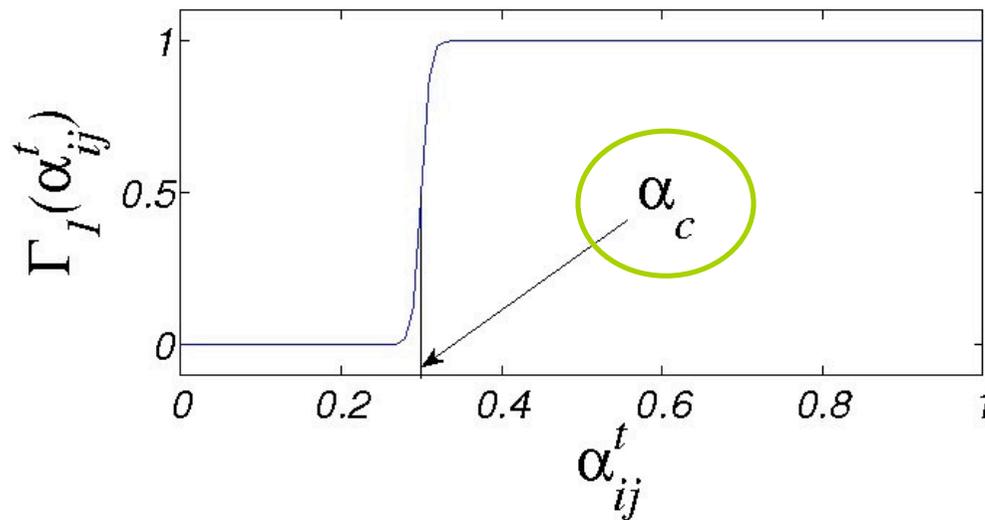
I. It seems plausible to assume that **individuals take into account the trustability** of a given source of information, when evaluating an external input.

II. This quantity, that we shall call **affinity, evolve dynamically** in time as follows the history of each individual.

III. The **affinity can be modified (enhanced/depressed)** so to resolve the tension that one faces when challenged by a conflicting viewpoint (**resolve the dissonance**).

# Accounting for mutual affinity

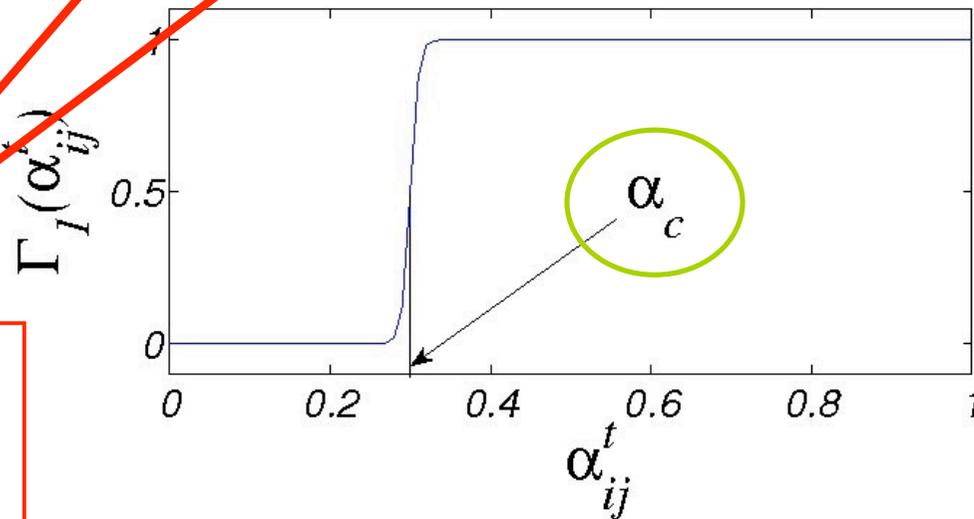
$$O_i^{t+1} = O_i^t - \mu \Delta O_{ij}^t \Gamma_1(\alpha_{ij}^t)$$



$$\Gamma_1(\alpha_{ij}^t) = \frac{1}{2} [\tanh(\beta_1(\alpha_{ij}^t - \underline{\alpha_c})) + 1]$$

# Accounting for mutual affinity

$$O_i^{t+1} = O_i^t - \mu \Delta O_{ij}^t \Gamma_1(\alpha_{ij}^t)$$

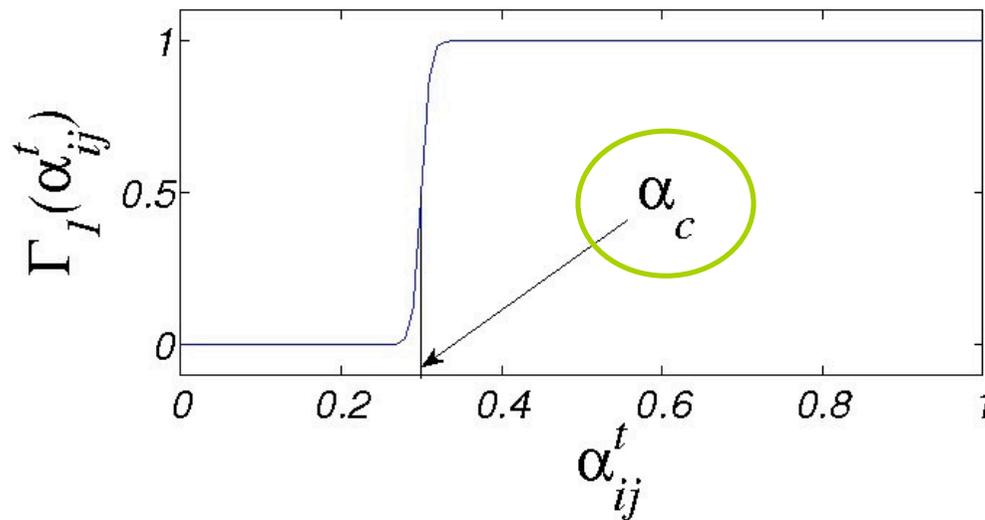


As in  
Deffuant's  
model

$$\Gamma_1(\alpha_{ij}^t) = \frac{1}{2} [\tanh(\beta_1(\alpha_{ij}^t - \underline{\alpha_c})) + 1]$$

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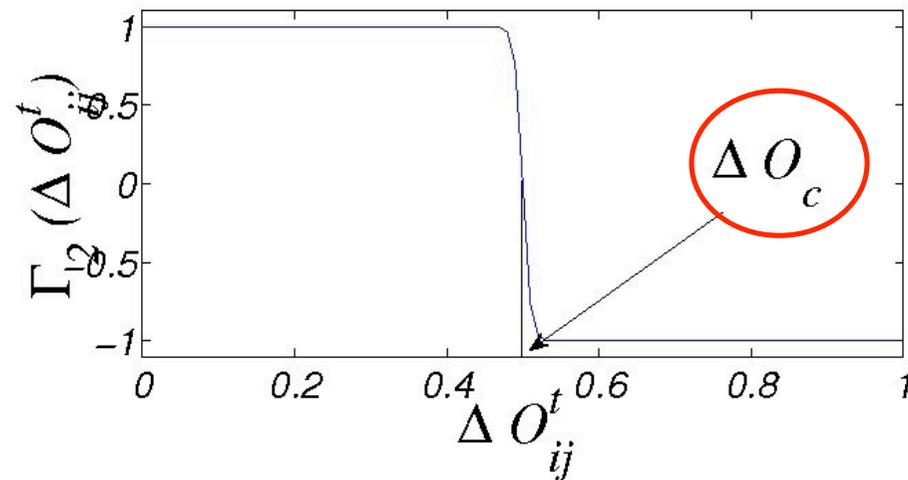


NxN  
affinity matrix

$$\Gamma_1(\alpha_{ij}^t) = \frac{1}{2} [\tanh(\beta_1(\alpha_{ij}^t - \underline{\alpha_c})) + 1]$$

# Accounting for mutual affinity

$$\alpha_{ij}^{t+1} = \alpha_{ij}^t + \alpha_{ij}^t [1 - \alpha_{ij}^t] \Gamma_2 (\Delta O_{ij})$$



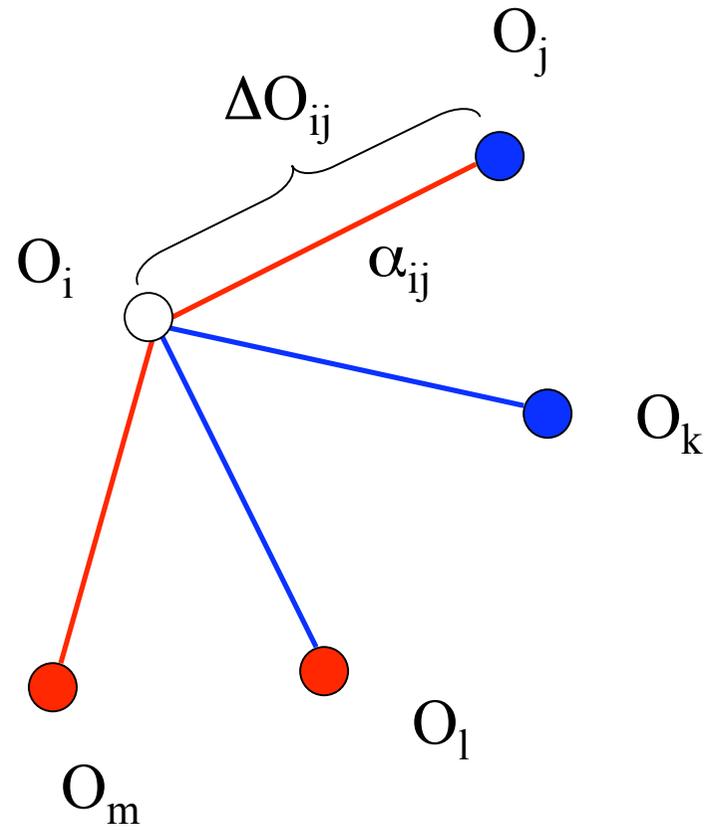
$$\Gamma_2 (\Delta O_{ij}) = -\tanh(\beta_2(|\Delta O_{ij}^t| - \underline{\Delta O_c}))$$

$\Delta O \approx 1$  ● (different)

$\approx 0$  ● (similar)

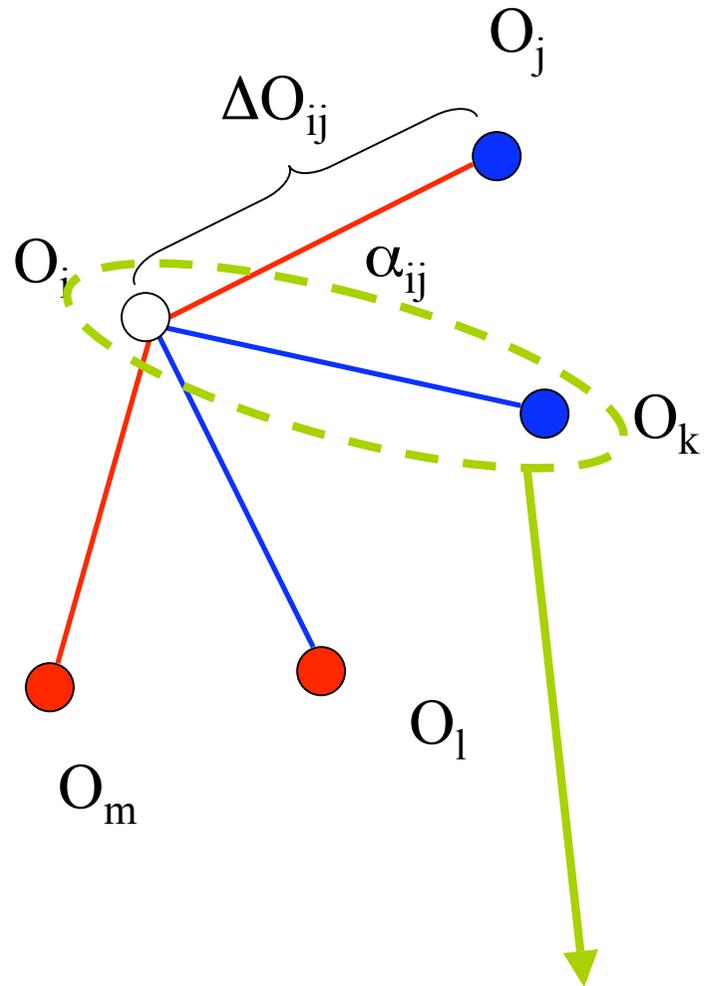
$\alpha_{ij} \approx 1$  — (more affine)

$\approx 0$  — (less affine)



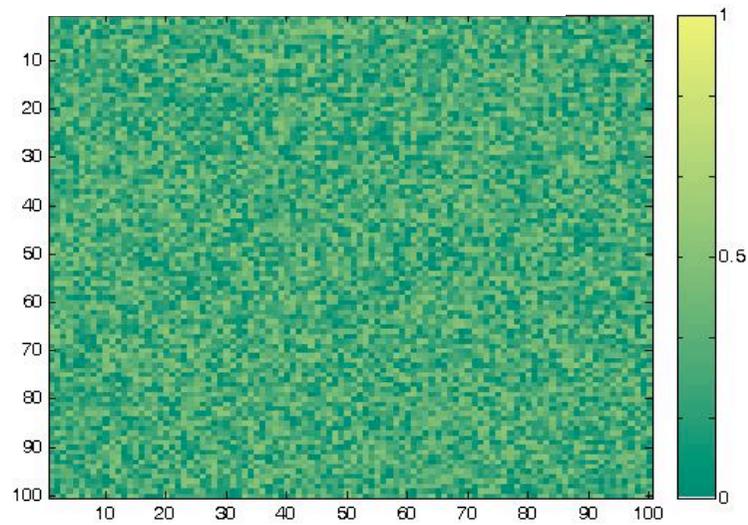
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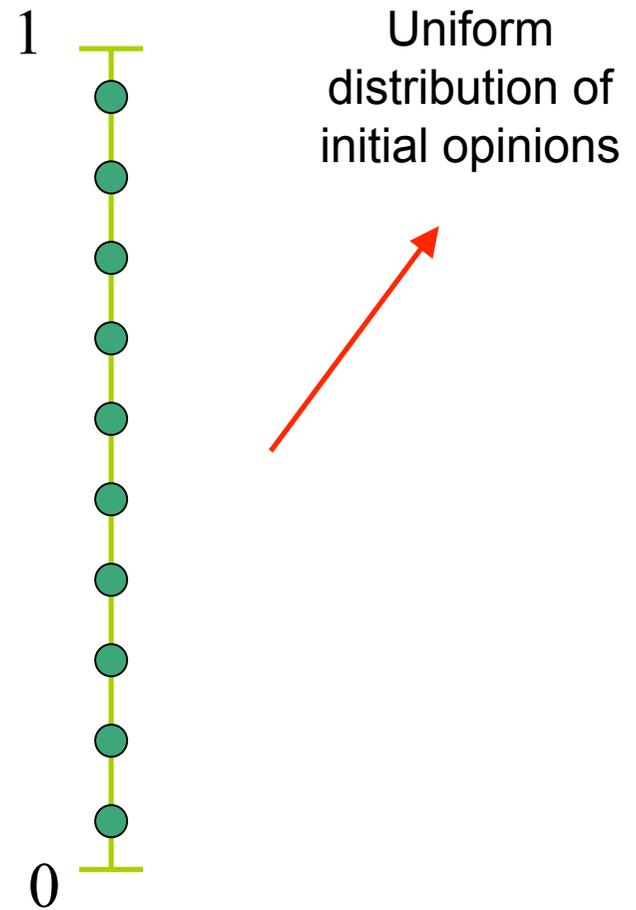


Individual k is the  
*ideal partner* of  
 agent i

# Initialising the system



Affinity matrix



1D opinion space

# The selection rule: social temperature

I.  $d_{ij}^t = \Delta O_{ij}^t (1 - \alpha_{ij}^t) \quad j = 1, \dots, N \quad j \neq i.$

Distance among agents  
i and j In *social space*

Random Gaussian noise  
to represent *social mixing*

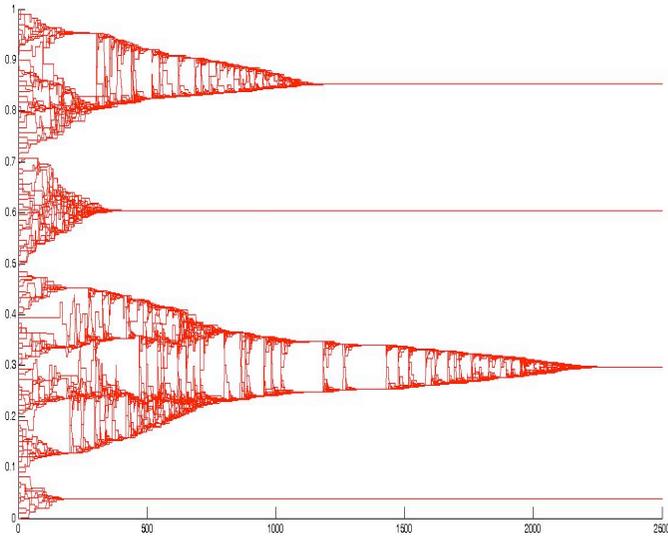
II.  $D_{ij}^\eta = d_{ij}^t + \eta_j(0, \sigma)$

● Random selection of individual  $i$

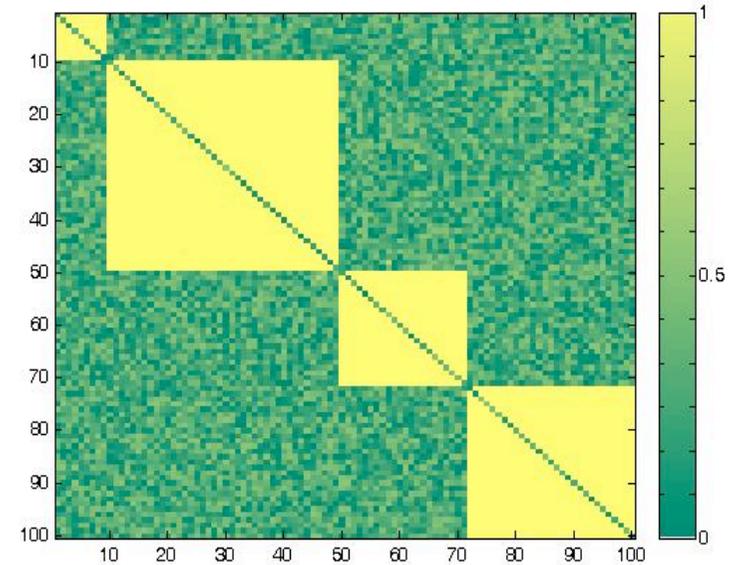
● Agent  $j$  follows from

$$\min_j D_{ij}^\eta$$

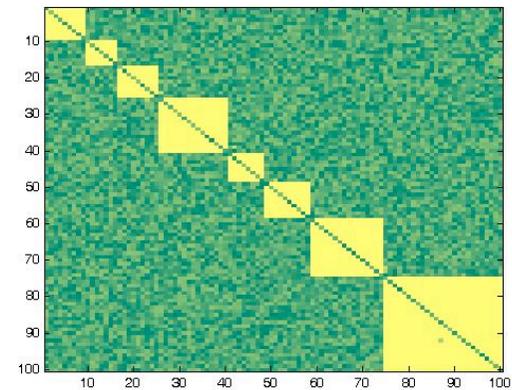
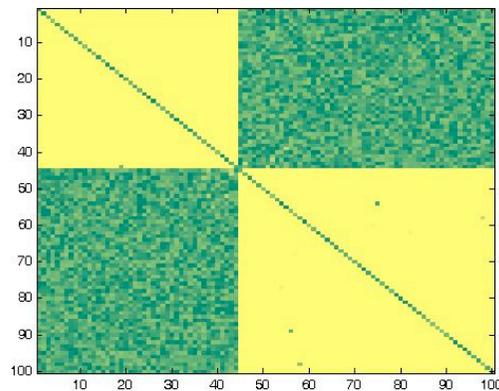
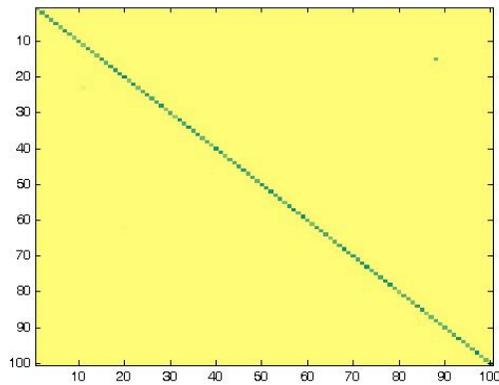
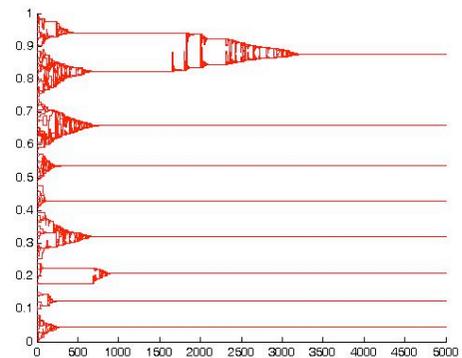
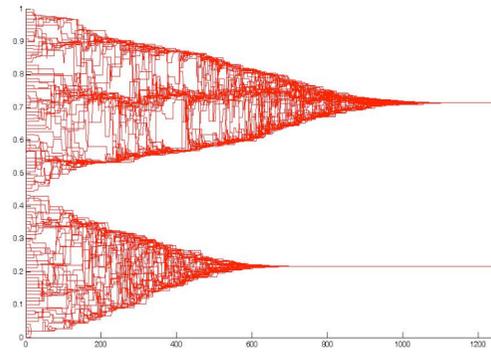
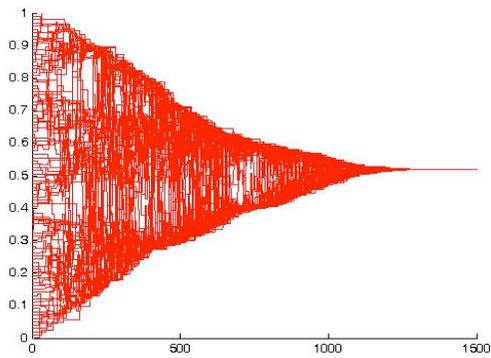
Evolution of opinion



Affinity Matrix

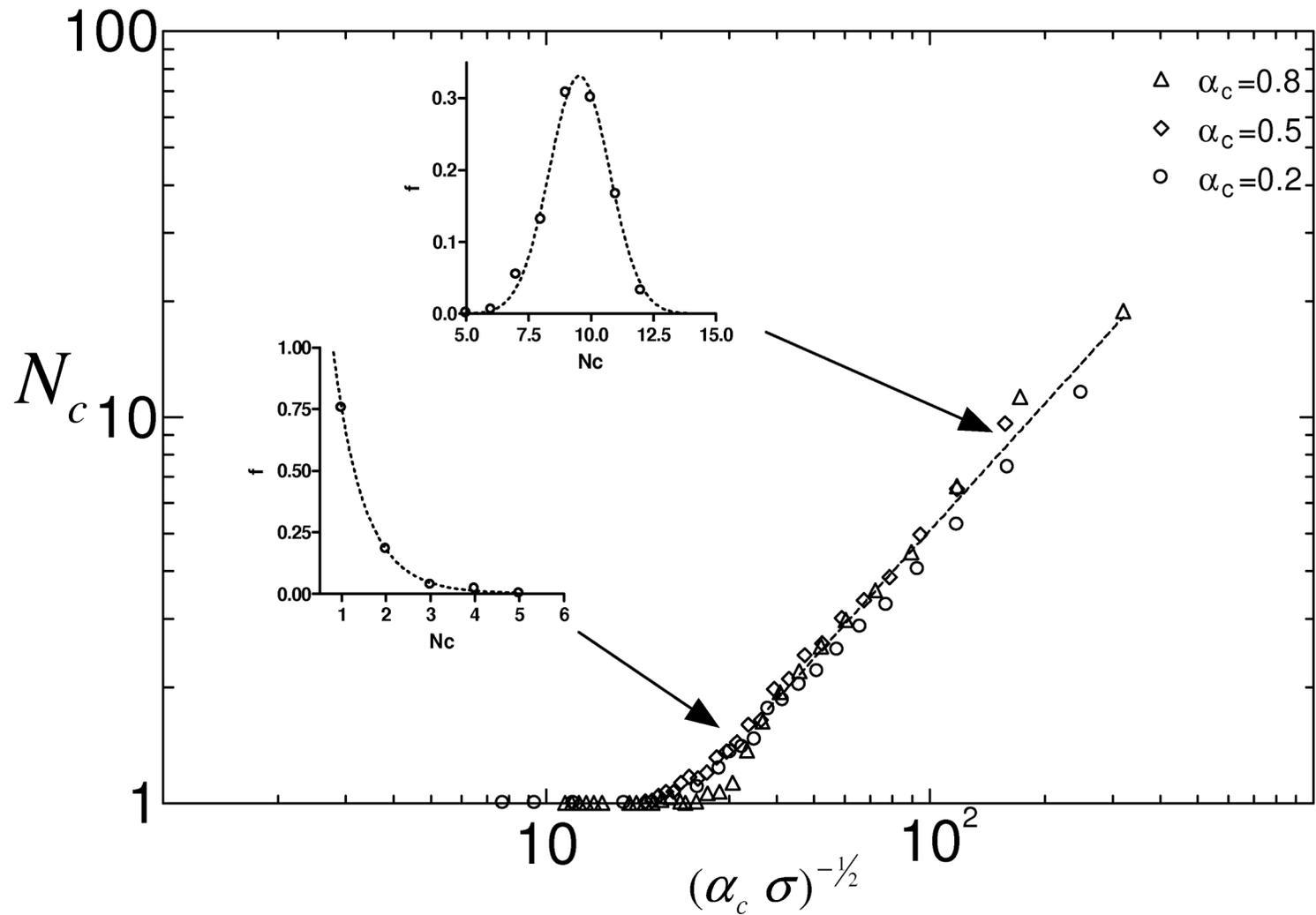


## A. Polarization vs. fragmentation

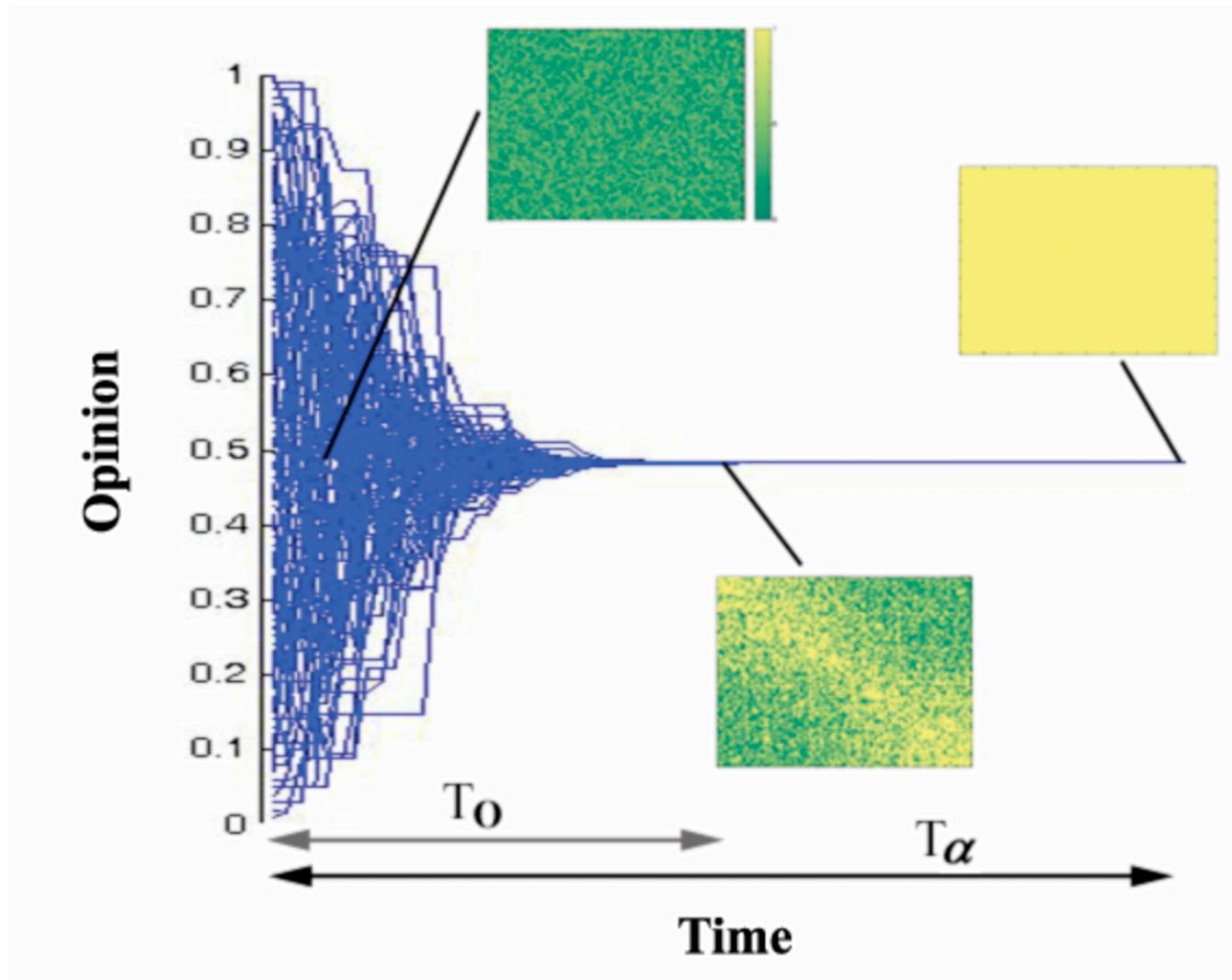


Decreasing mixing (social temperature)

Phase transition: from one to several (*metastable*) clusters



## B. On the convergence time: $T_0$ vs. $T_\alpha$

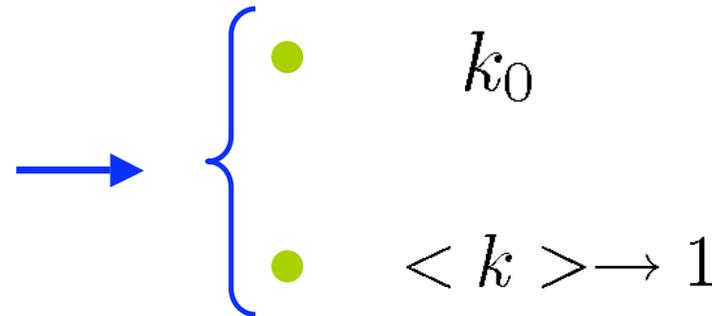


## Towards a characterization of $T_\alpha$ ...

The **affinity matrix** measures the degree of connection among agents  
(**weighted network**)

AVERAGE CONNECTIVITY

$$\langle k \rangle = \frac{1}{N(N-1)} \sum_{i,j} \alpha_{i,j}$$



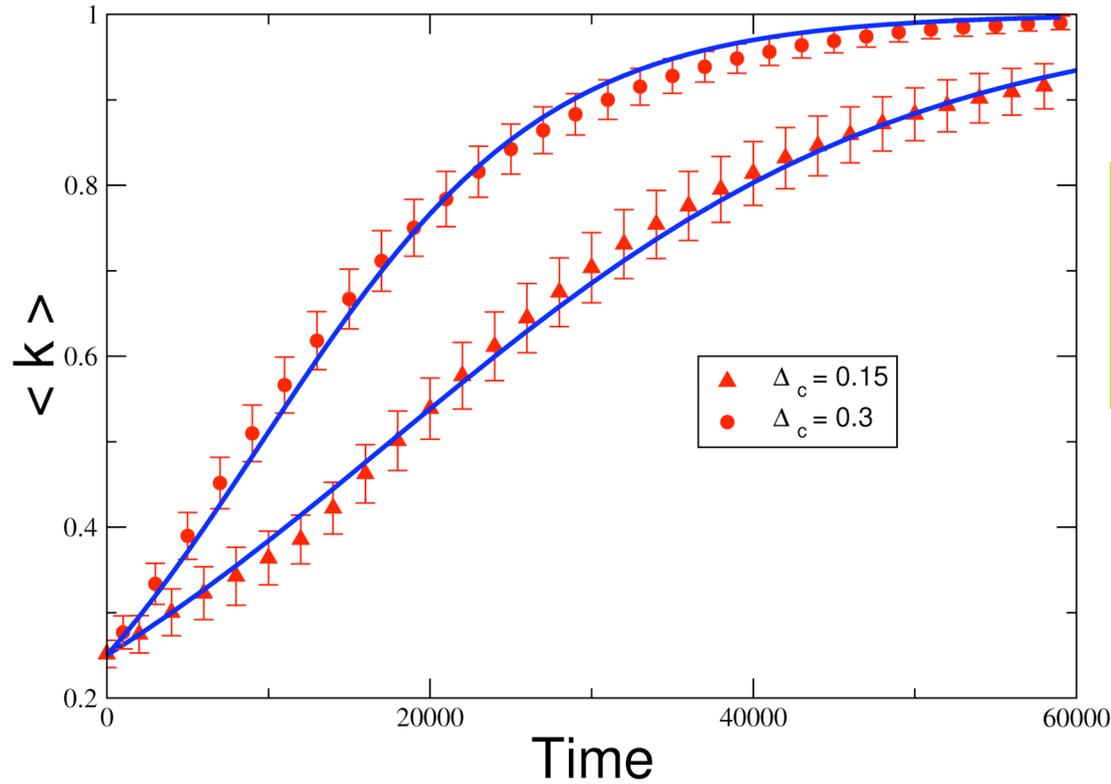
Accessing an estimate of  $T_\alpha$   
by monitoring the time evolution of  $\langle k \rangle$

DERIVING:

$$\frac{d \langle k \rangle}{dt} = \frac{1}{N(N-1)} \sum_{i,j} \frac{d\alpha_{i,j}}{dt}$$

FOLLOWS FROM SYSTEM DYNAMICS

if  $\sigma_\alpha^2 \ll 1$  then  $\frac{d \langle k \rangle}{dt} = \Gamma (\langle k \rangle - \langle k \rangle^2)$

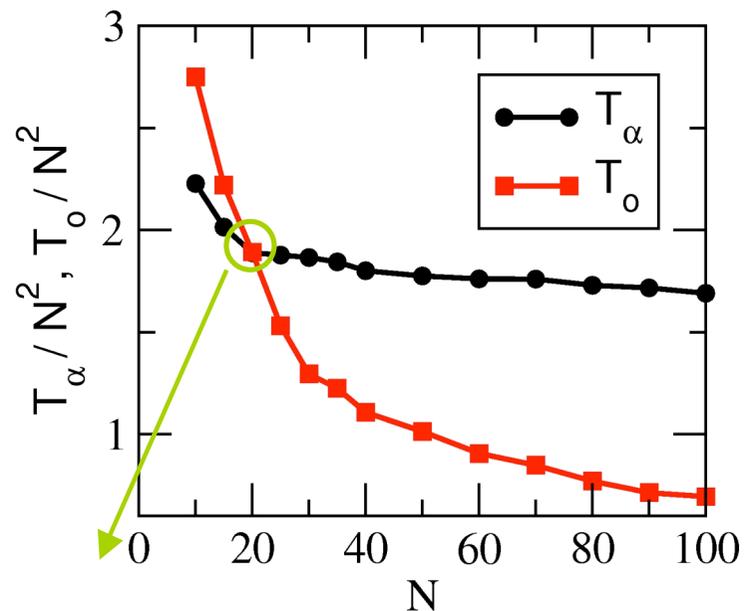


$$\langle k \rangle = \frac{1}{1 + \frac{1-k_0}{k_0} \exp(-\Gamma t)}$$

For large enough N:

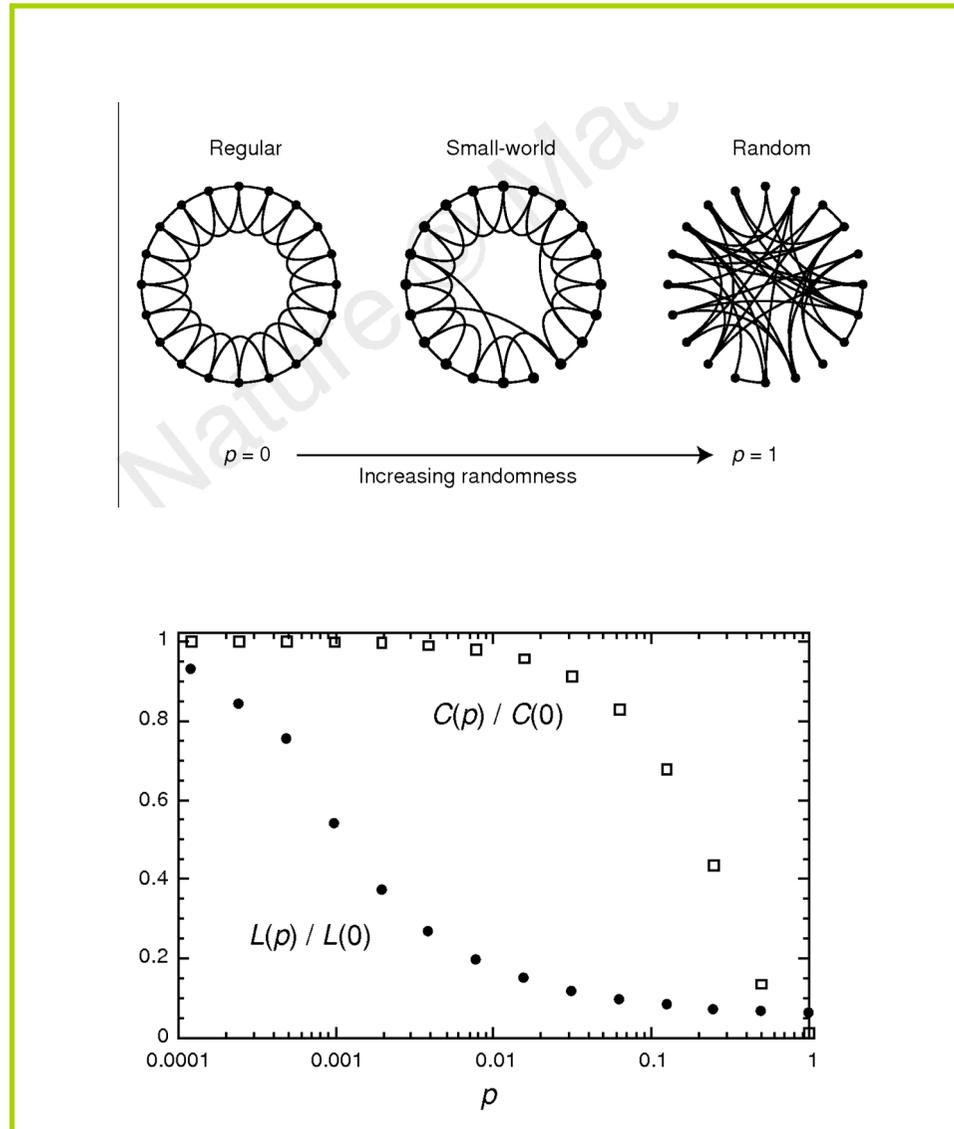
$$T_\alpha \propto N^2$$

Existence of a **critical group size**, a mechanism that is reported in the psychological literature (Bjorn). In a small group people get to know each other **before** converging to a final opinion, as opposed to the large group case.

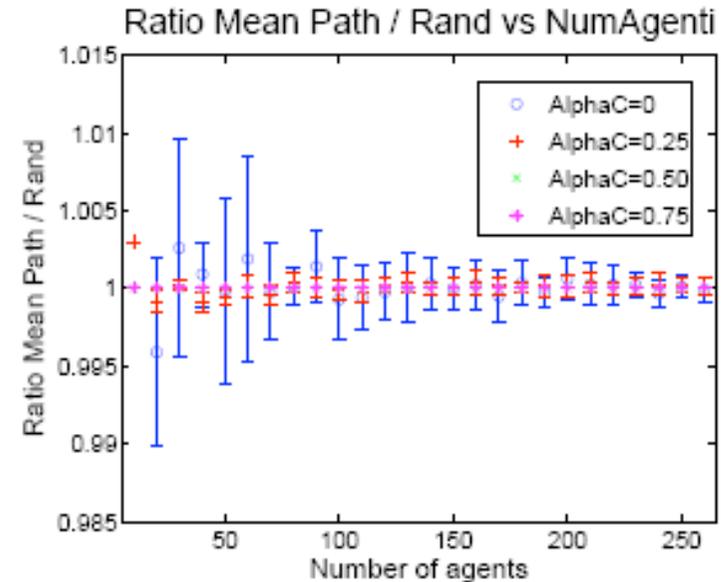


Crossover value **predicted**  
by the theory for  $N \sim 20$

## C. Emerging topology of the social networks



Watts and Strogatz, Nature 1998

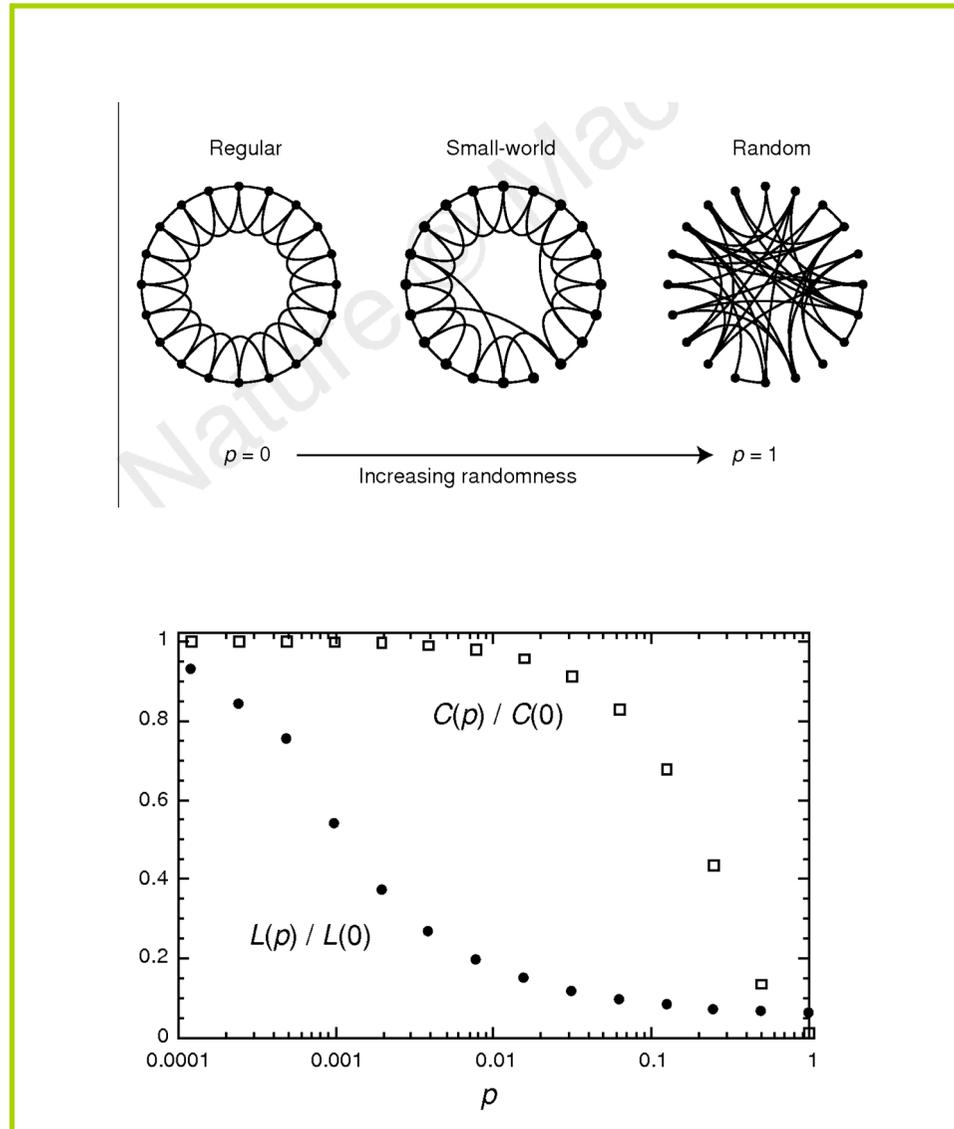


$$L = \frac{1}{N(N-1)} \sum_{i,j \in \mathcal{N}, i \neq j} d_{ij}$$

$d_{ij}$  = length of geodesic path between  $i$  and  $j$

$a_{ij} = [\alpha_{ij} - \alpha_f]$  adjacency matrix

## C. Emerging topology of the social networks

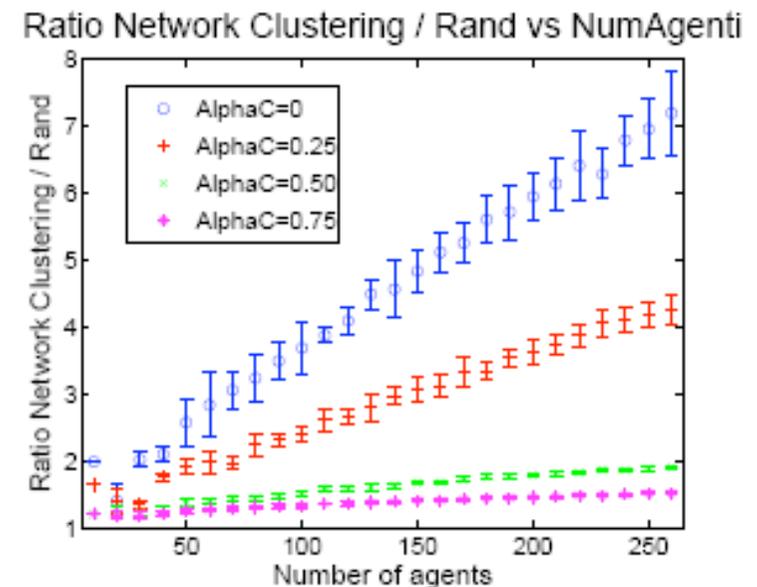


Watts and Strogatz, Nature 1998

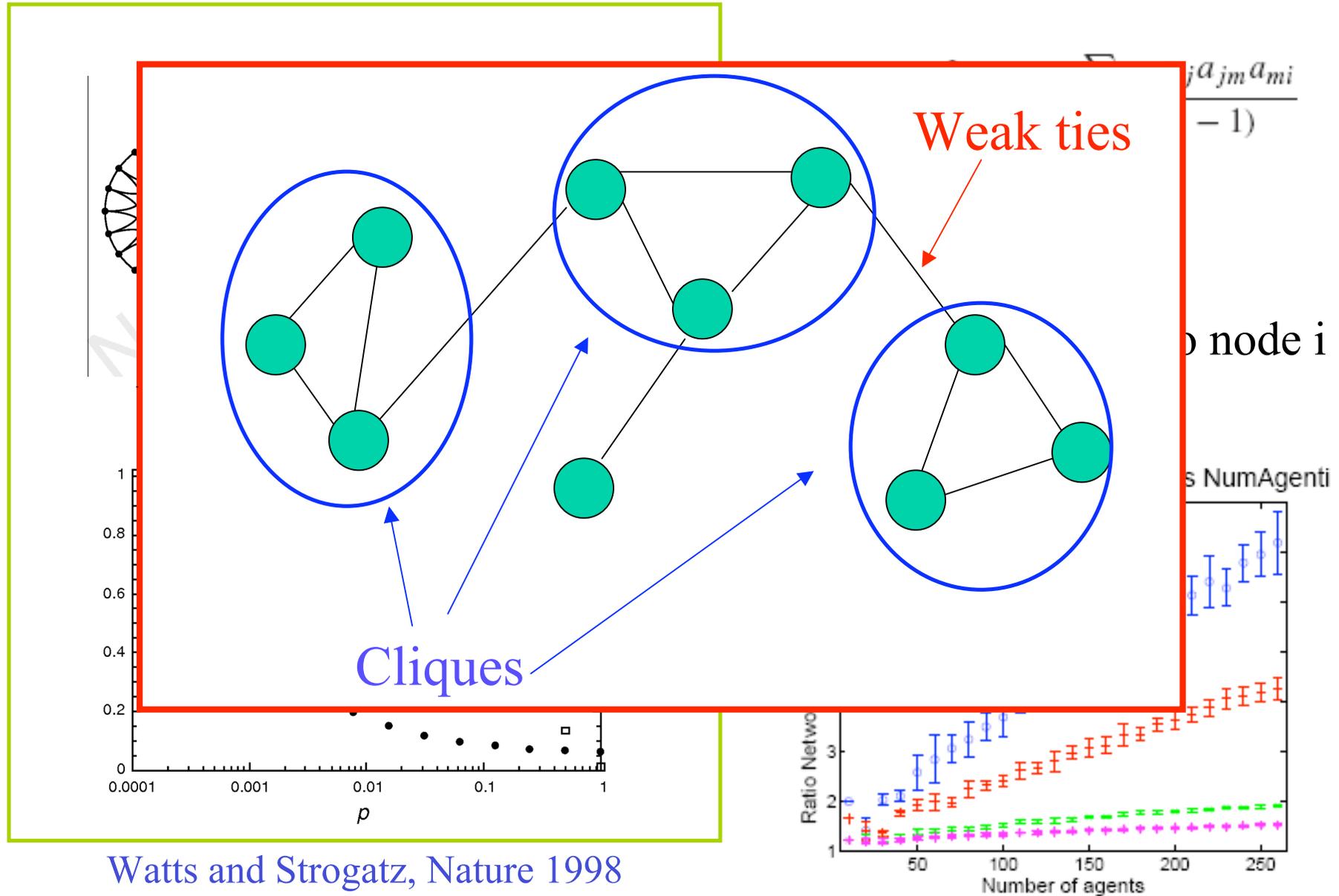
$$c_i = \frac{2e_i}{k_i(k_i - 1)} = \frac{\sum_{j,m} a_{ij}a_{jm}a_{mi}}{k_i(k_i - 1)}$$

$$C = \langle c \rangle = \frac{1}{N} \sum_{i \in \mathcal{N}} c_i$$

$e_i$  = nodes adjacent to node  $i$



# C. Emerging topology of the social networks



Watts and Strogatz, Nature 1998

## Summing up.....

### PRO

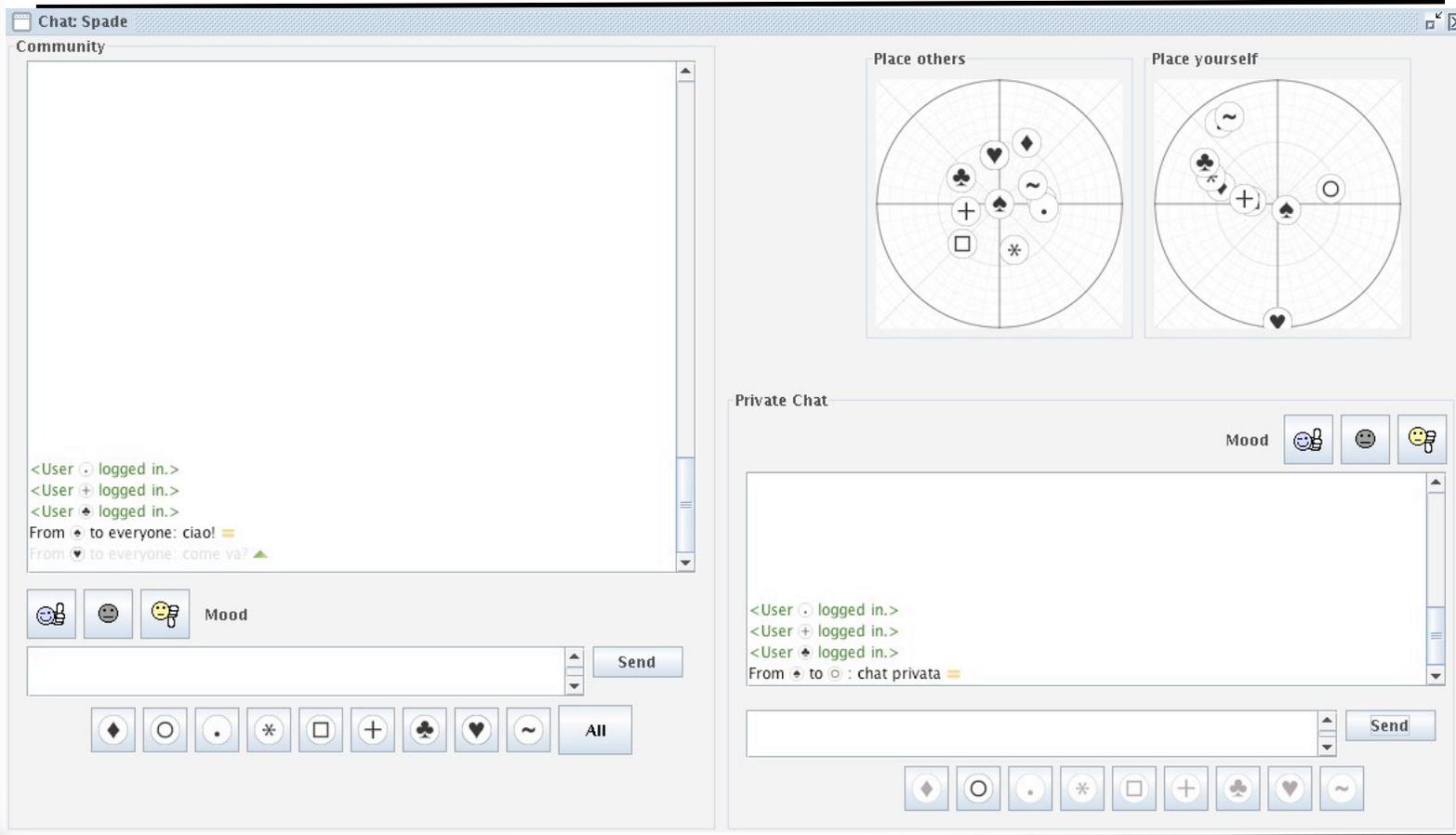
- I. Models do exist that enable us to study the opinion formation process
- II. Consensus is an emerging feature, as opposed to fragmentation
- III. Models can incorporate specific psychological aspect within a self-consistent dynamical framework

### CONS

- A. Too high degree of arbitrariness in setting up the models
- B. Need for experimental inputs that allows to constraint the model
- C. The level of model complexity has to be tuned so to reflect the richness of the experimental inputs

# The virtual community experiment (Andrea Guazzini et al.)

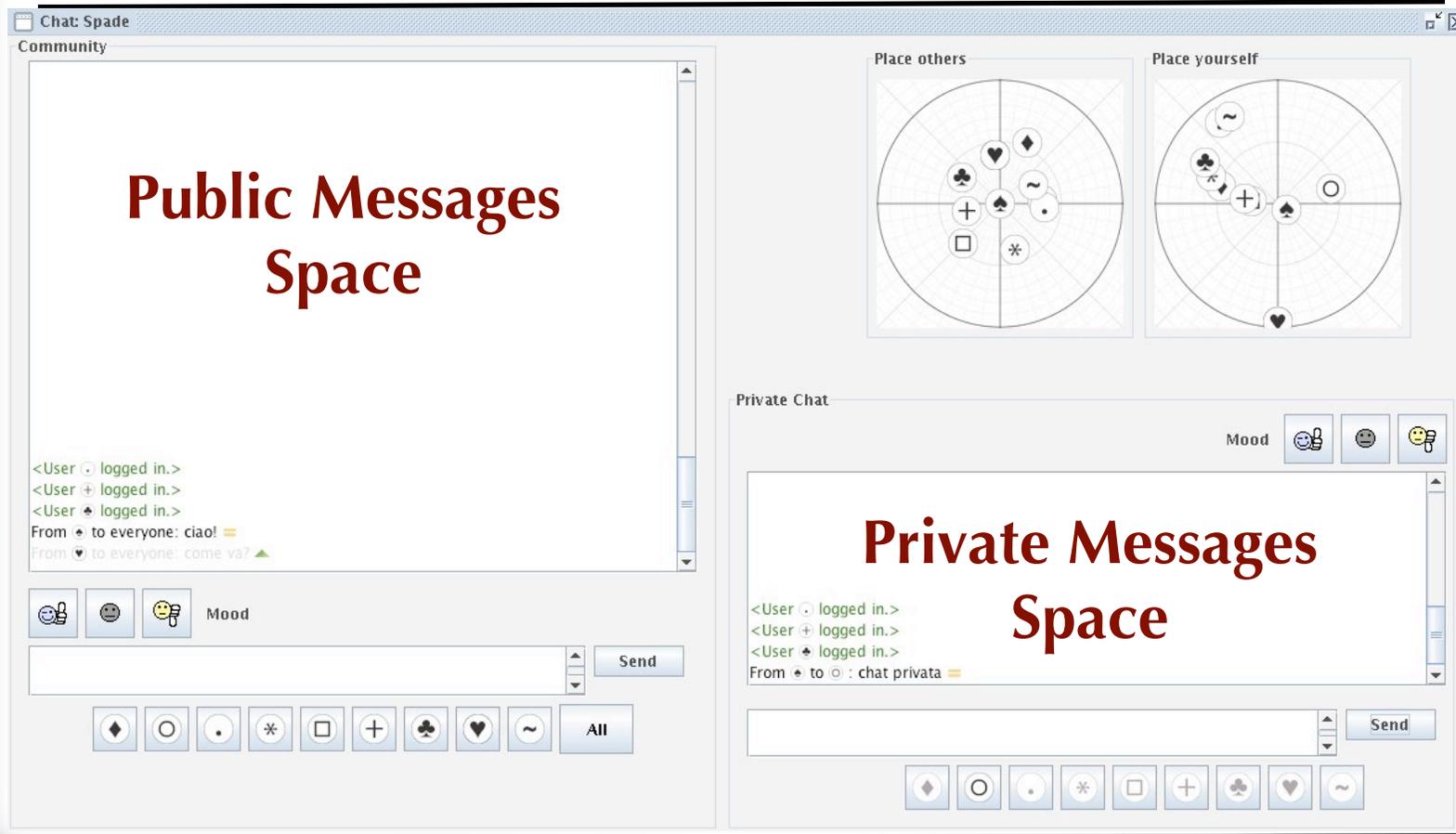
## The chat interface



Cognitive network dynamics in chatlines

# The virtual community experiment (Andrea Guazzini et al.)

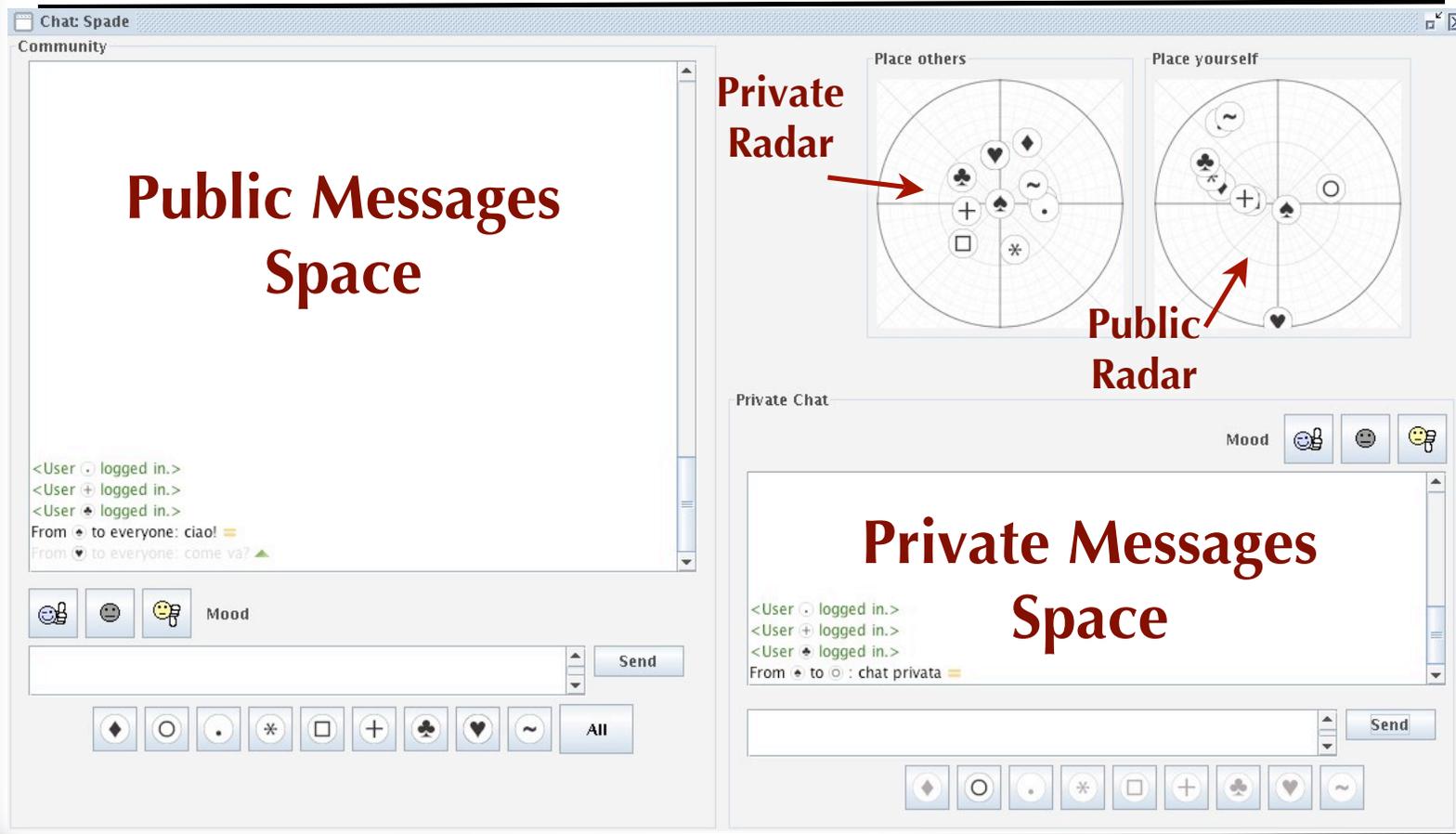
## The chat interface



Cognitive network dynamics in chatlines

# The virtual community experiment (Andrea Guazzini et al.)

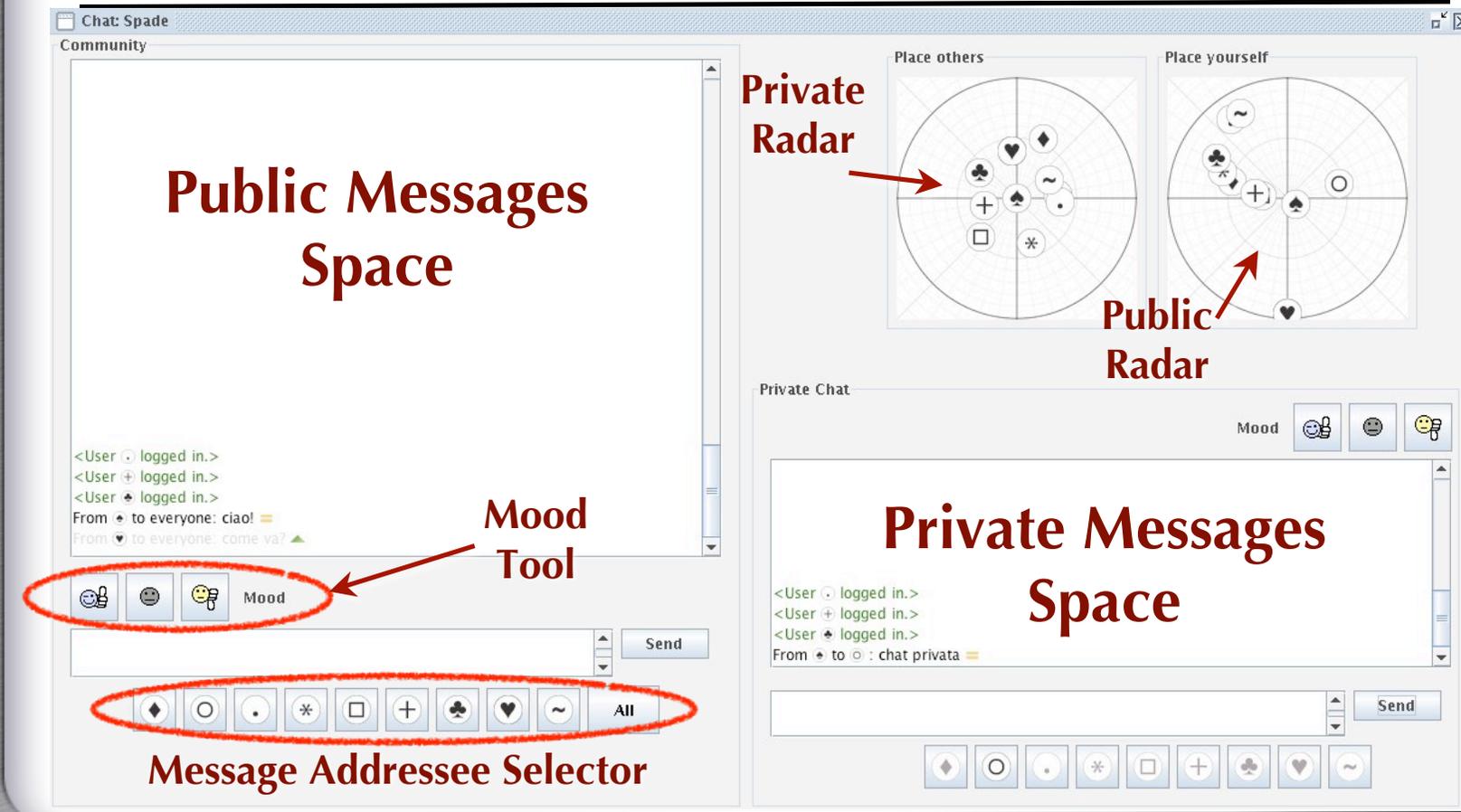
## The chat interface



Cognitive network dynamics in chatlines

# The virtual community experiment (Andrea Guazzini et al.)

## The chat interface



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