

# Modeling biomolecular networks: from metabolism and its regulation to protein-protein interactions

Sergei Maslov

Brookhaven National Laboratory



# What is a complex system?

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- **Complex systems** have **many interacting components** ( $10^{11}$  neurons,  $10^4$  types of proteins,  $10^6$  routers,  $10^9$  web pages)
- All components are **different from each other**
- **Systems** traditionally studied by **physics** also have many interacting components ( $10^{23}$  electrons in a superconductor)
- But they are **all the same!**



# Networks in complex systems

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- Since components are different, first question: **which pairs directly interact?**
- The answer can be visualized as a **network**
- Network is the **backbone** of the underlying **complex system**
- In my talk I will first propose a model of evolution of the **backbone** (Part 1) and then **put dynamics** on it (Part 2)

# Part 1: Parkinson's law in biology

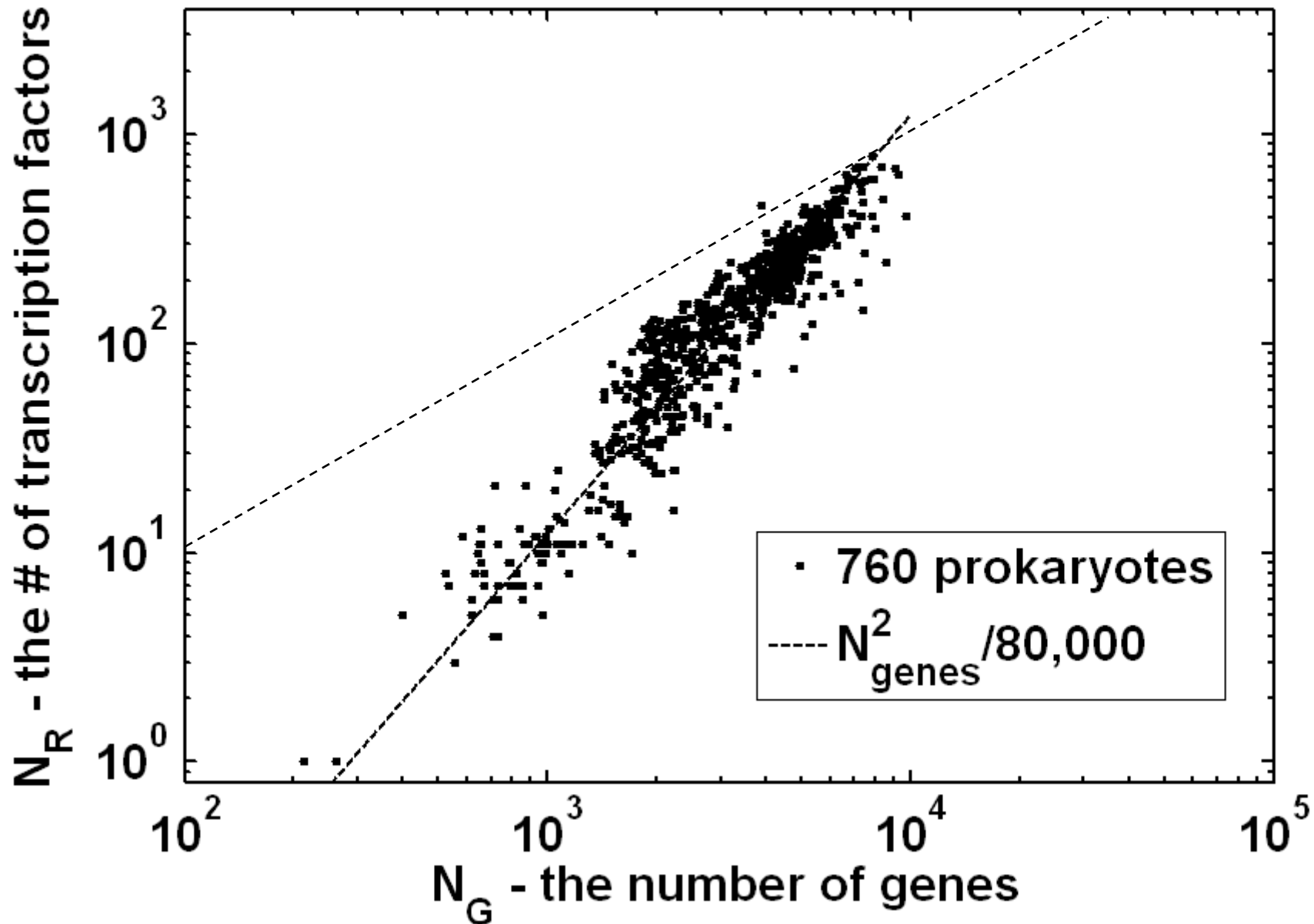


## Parkinson's Law

**The report of the Royal Commission on the Civil Service was published on Thursday afternoon. Time has not permitted any comment in this week's issue of The Economist on the contents of the Report. But the startling discovery enunciated by a correspondent in the following article is certainly relevant to what should have been in it.**

Nov 19th 1955 | From *The Economist* print edition

Stover *et al.*, Nature (2000)    van Nimwegen, TIG (2003)





# Let's play with this scaling law

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- $N_R = N_G^2 / 80,000 \rightarrow \Delta N_R = \Delta N_G \cdot 2N_G / 80,000$
- When a new regulated function is added  $\Delta N_R = +1$ ,  
 $\Delta N_G / \Delta N_R = 40,000 / N_G$ 
  - $\sim 40$  new genes per function for  $N_G = 1000$
  - $\sim 4$  new genes (1 regulator + 3 non-regulatory genes)  
for the largest bacterial genomes with  $N_G \sim 10,000$
- One needs to explain **why  $\Delta N_G / \Delta N_R$  systematically decreases with genome size as  $1/N_G$**

# “Home Depot” or toolbox model



## Toolbox model of evolution of prokaryotic metabolic networks and their regulation

Sergei Maslov<sup>a,1</sup>, Sandeep Krishna<sup>b</sup>, Tin Yau Pang<sup>a,c</sup>, and Kim Sneppen<sup>b</sup>

<sup>a</sup>Department of Condensed Matter Physics and Materials Science, Brookhaven National Laboratory, Upton, NY 11973; <sup>b</sup>Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark; and <sup>c</sup>Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794-3800

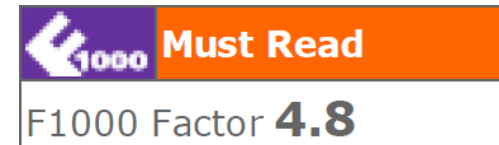
Edited by David J. Lipman, National Institutes of Health, Bethesda, MD, and approved April 16, 2009 (received for review March 23, 2009)

It has been reported that the number of transcription factors encoded in prokaryotic genomes scales approximately quadratically with their total number of genes. We propose a conceptual explanation of this finding and illustrate it using a simple model in which metabolic and regulatory networks of prokaryotes are

A simple evolutionary model explains both these empirical observations in a unified framework based on modular functional design of prokaryotic metabolic networks and their regulation.

### Toolbox View of Metabolic Networks

Disclaimer: authors of this study (unfortunately) received no financial support from Home Depot, Inc. Homebase, LTD or Obi, GMBH

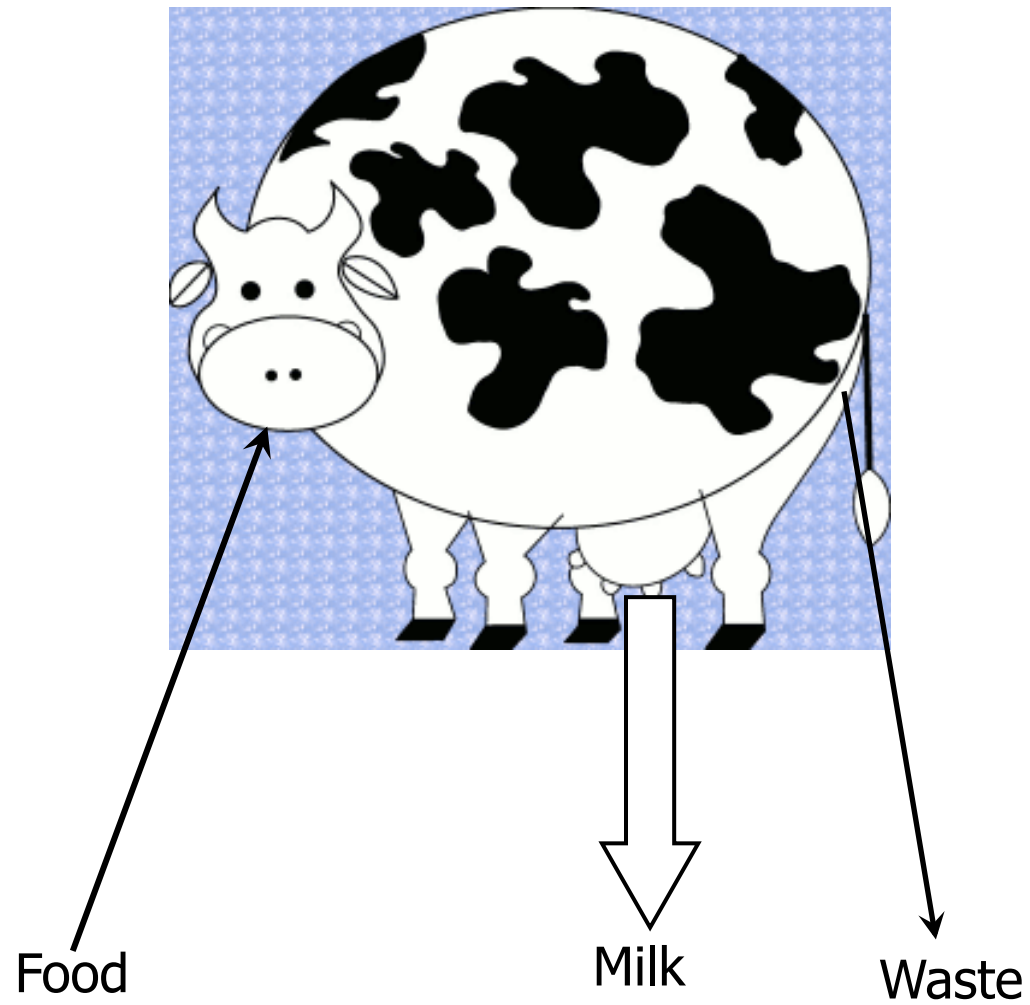


# “Home Depot” argument

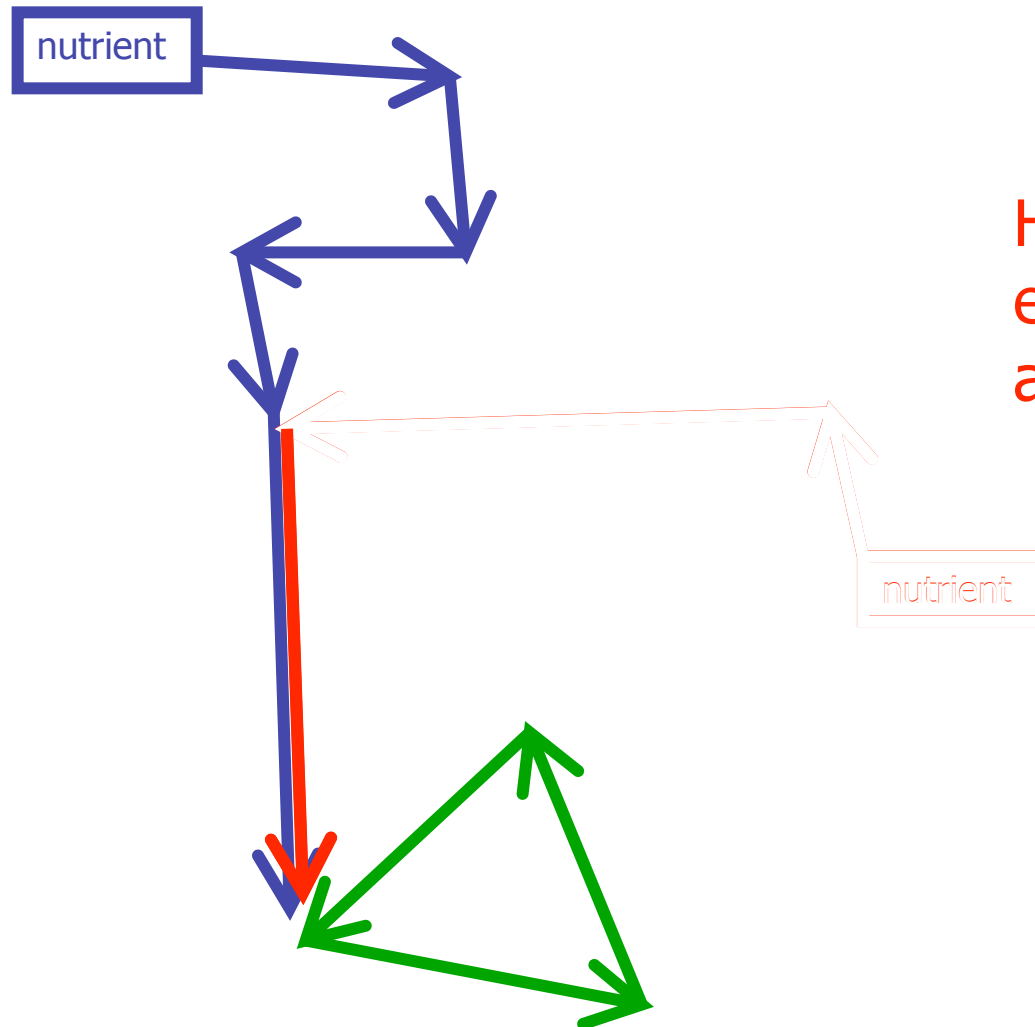
- Inspired by personal experience as a new **homeowner**
- **Tools** are bought to **accomplish functional tasks** e.g. fix a leaking faucet
- **Duplicate tools are returned** to “Home Depot”
- As your toolbox grows you need to get fewer and fewer new tools to accomplish a new task
- **Bacteria** also have tools encoded by **non-regulatory “workhorse” genes** (e.g. for metabolic enzymes)
- **Entire pathways** (collections of tools) are acquired from other bacteria by **Horizontal Gene Transfer**
- **Regulators** control these **pathways** (we assume one regulator per task/pathway)
- **Redundant genes** are promptly **deleted** (in prokaryotes)
- As the genome gets larger you need fewer new genes per new regulated function – **FASTER THAN LINEAR SCALING**



# Spherical cow model of metabolic networks



Pathways could be also removed



Horizontal gene transfer:  
entire pathways could be  
added in one step

Central metabolism → anabolic  
pathways → biomass

- New functions are added by Horizontal Gene Transfer of entire pathways (collections of tools)
- They come from the **universal network of size  $N_{\text{univ}}$**  composed of all reactions in all organisms (bacterial answer to “Home depot”)
- The current size of the toolbox ( $\sim$ # of genes  $\sim$  # of enzymes  $\sim$  # of metabolites):  $N_G$
- Probability to join the existing pathway:  $p_{\text{join}} = N_G / N_{\text{univ}}$
- $L_{\text{pathway}} = 1/p_{\text{join}} = N_{\text{univ}}/N_G$
- Assume one regulator per function/pathway:  $\Delta N_G / \Delta N_R = L_{\text{pathway}} = N_{\text{univ}}/N_G$
- Quadratic law:  $N_R = N_G^2 / 2N_{\text{univ}}$



# Different universal networks give the same result

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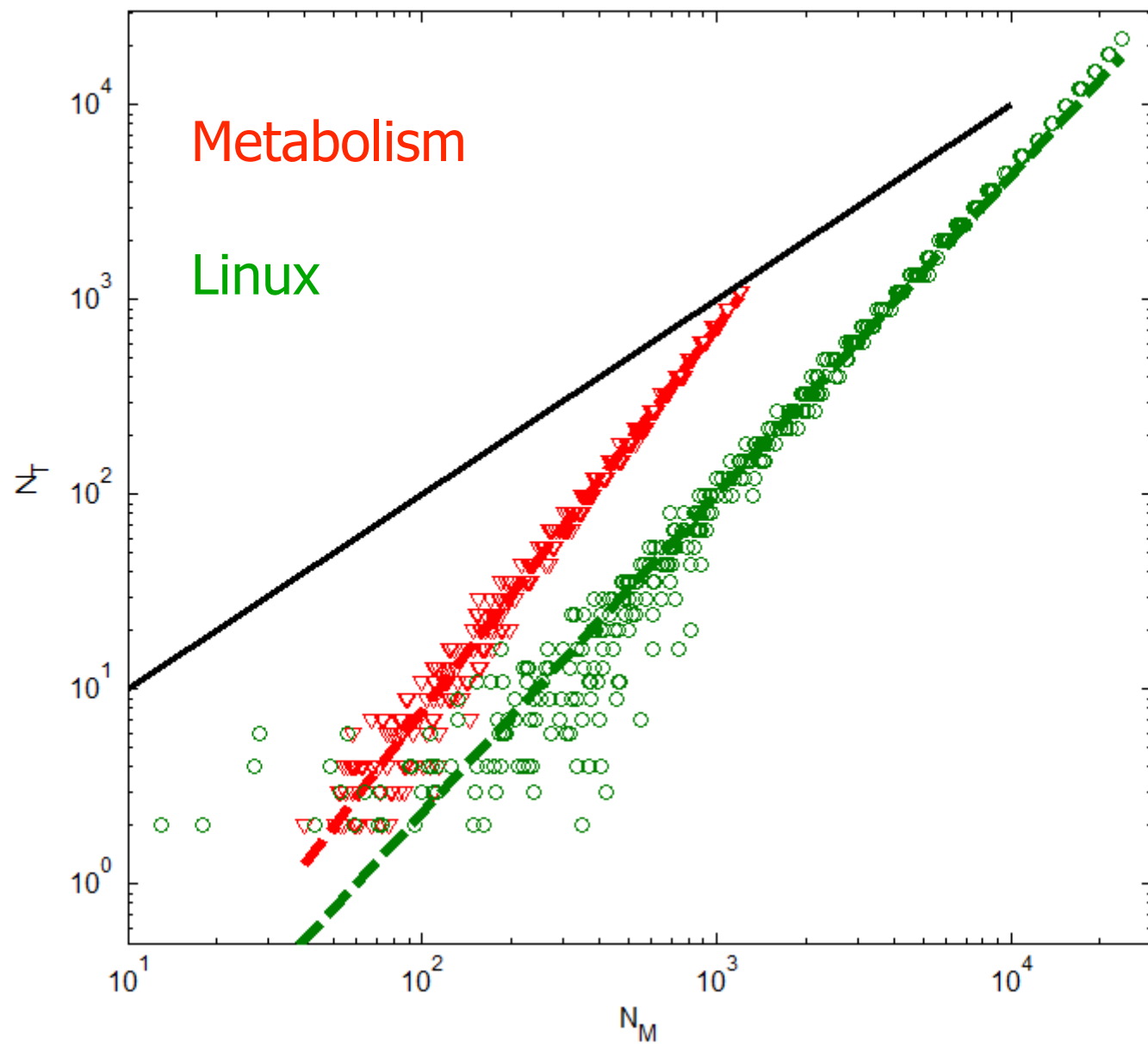
- **Random branched network:** analytically solved to give  $N_R \sim N_{\text{met}}^2$
- **Union of all metabolic reactions in the KEGG database:** numerically solved to give  $N_R \sim N_{\text{met}}^{2.0 \pm 0.3}$

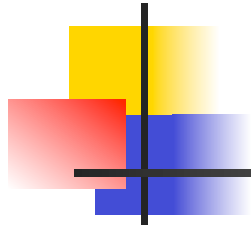


# “Home Depot” model is not limited to biology

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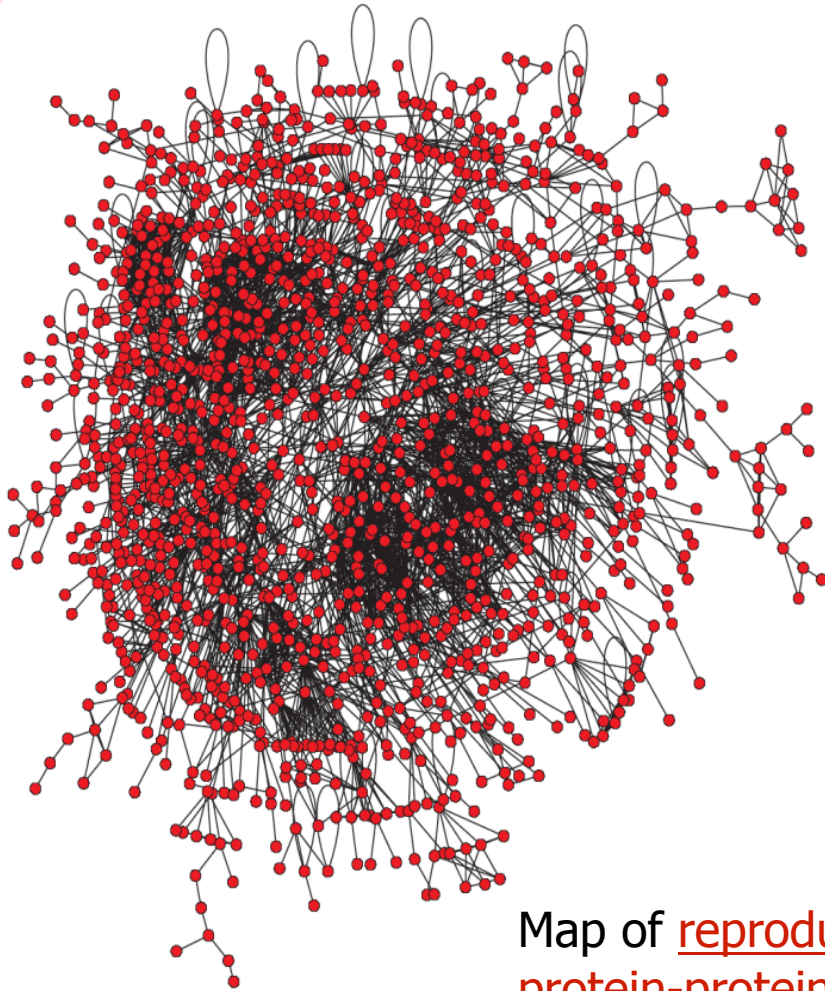
- Adding new units becomes easier as system grows:  
**ECONOMY OF SCALE**
- **Quadratic scaling** is expected in any **multi-level system** formed by **mutually-dependent units**
- **Software packages** installed in **Linux**  
 $N_{\text{usable functions}} \sim N_{\text{installed packages}}^{1.7 \pm 0.3}$
- Expected: in networks of interdependent **technological innovations** (e.g. patents using other patents), **supply networks** of companies





## Part 2: Mass Action Equilibrium in protein binding networks

# Small world of protein-protein interaction networks



- **>80%** of proteins are all connected **in one giant cluster** of PPI network
- **Small-world effect** median network distance – **6 steps**

Map of reproducible (>2 publications) protein-protein interactions in yeast





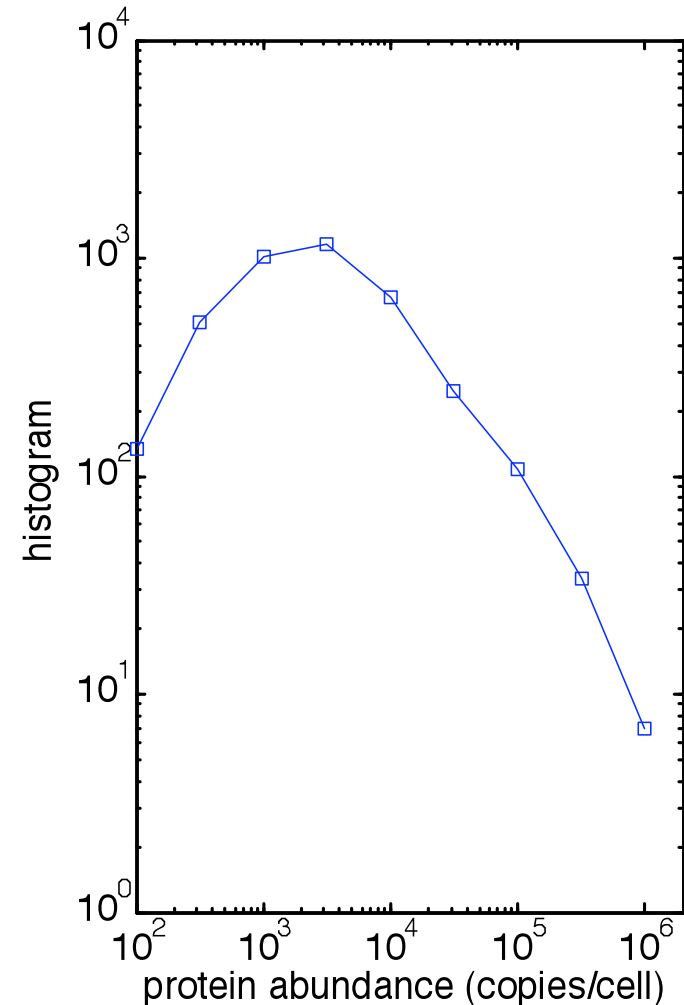
# Why small-world property might cause problems

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- Are small world networks more robust?
- **Internet** – more connected is better
- **Small world binding networks** could indiscriminately spread perturbations
  - **Systematic changes:** large deterministic changes in concentrations of a small number of proteins SM, I. Ispolatov, PNAS and NJP (2007)
  - **Noise:** small changes in concentrations of a large number of proteins K.-K. Yan, D. Walker, SM, PRL (2008)

# My “spherical cow” assumptions

- Protein concentrations  $C_i$  of all yeast proteins (under the rich growth medium conditions) and subcellular localizations are experimentally known (group of Weissman @ UCSF)
- Consider only reproducible independently confirmed protein-protein interactions for non-catalytic binding (kinase-substrate pairs ~5%)
- The network: ~4000 heterodimers and ~100 multi-protein complexes (we assume no cooperative binding in complexes) connecting ~1700 proteins
- Know the relevant average of dissociation constants  $K_{ij} \sim 10\text{nM}$ . Turned out their distribution around this average DOES NOT MATTER MUCH!!!
- Use “evolutionary motivated” binding strength:  $K_{ij} = \max(C_i, C_j) / \text{const}$ , which is sufficient to bind considerable fraction of two proteins in a heterodimer





# Law of Mass Action (LMA)

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- $dD_{AB}/dt = r^{(on)}_{AB} F_A F_B - r^{(off)}_{AB} D_{AB}$

- In the equilibrium:

$$D_{AB} = F_A F_B / K_{AB}; \quad C_A = F_A + D_{AB}; \quad C_B = F_B + D_{AB}$$

or  $F_A = C_A / (1 + F_B / K_{AB})$  and  $F_B = C_B / (1 + F_A / K_{AB})$

- In a **network**:  
A system of  $\sim 2000$   
**nonlinear equations**  
for  $F_i$  that can be  
**solved only numerically**

$$F_i = \frac{C_i}{1 + \sum_{j \text{ nni.}} \frac{FK_{jij}}$$



# Propagation of perturbations: the *in silico* study

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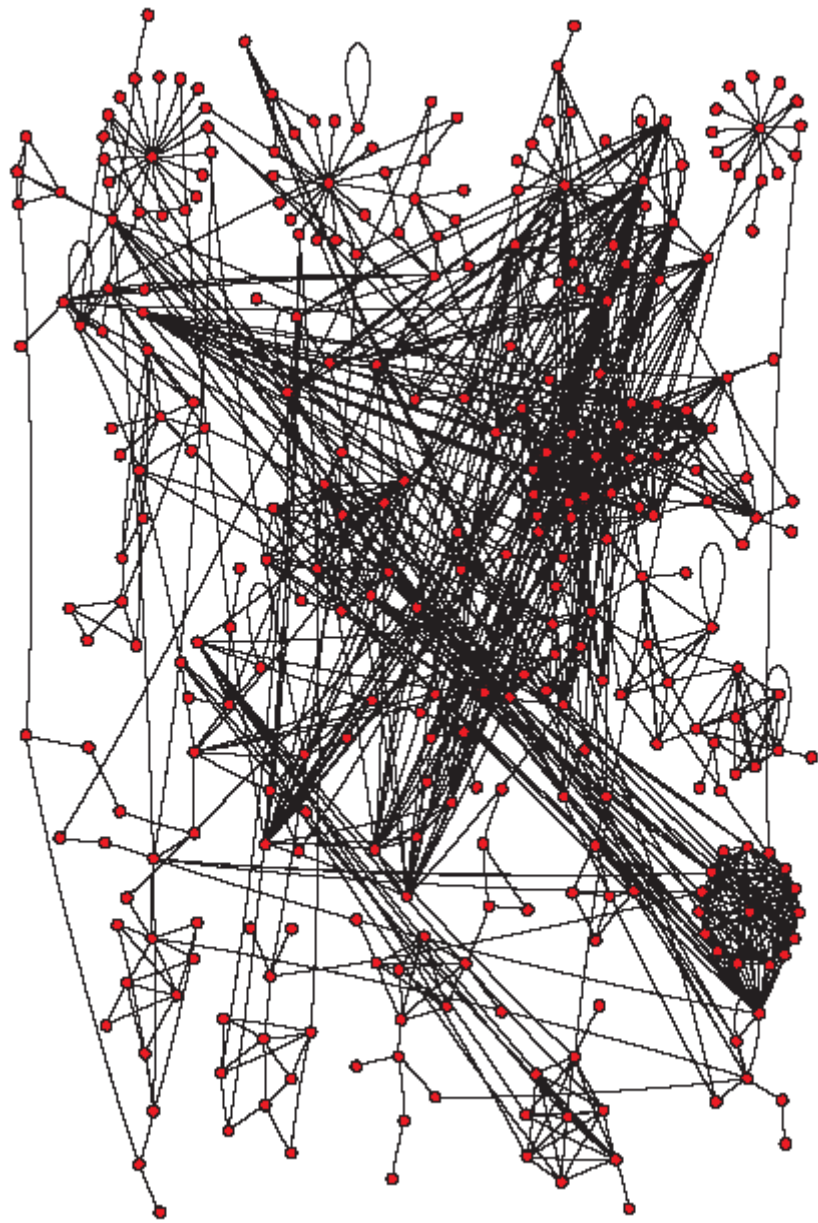
- Calculate the unperturbed **mass action equilibrium**
- Simulate a **twofold increase** of the concentration  $C_A \rightarrow 2C_A$  of just **one type of protein** and **recalculate equilibrium free concentrations**  $F_i$  of all other proteins
- Look for **cascading perturbations**:  
 $A \rightarrow B \rightarrow C \rightarrow D$  with sign-alternation:  
 $A$  ( $\uparrow$  **up**),  $B$  ( $\downarrow$  **down**),  $C$  ( $\uparrow$  **up**),  $D$  ( $\downarrow$  **down**)

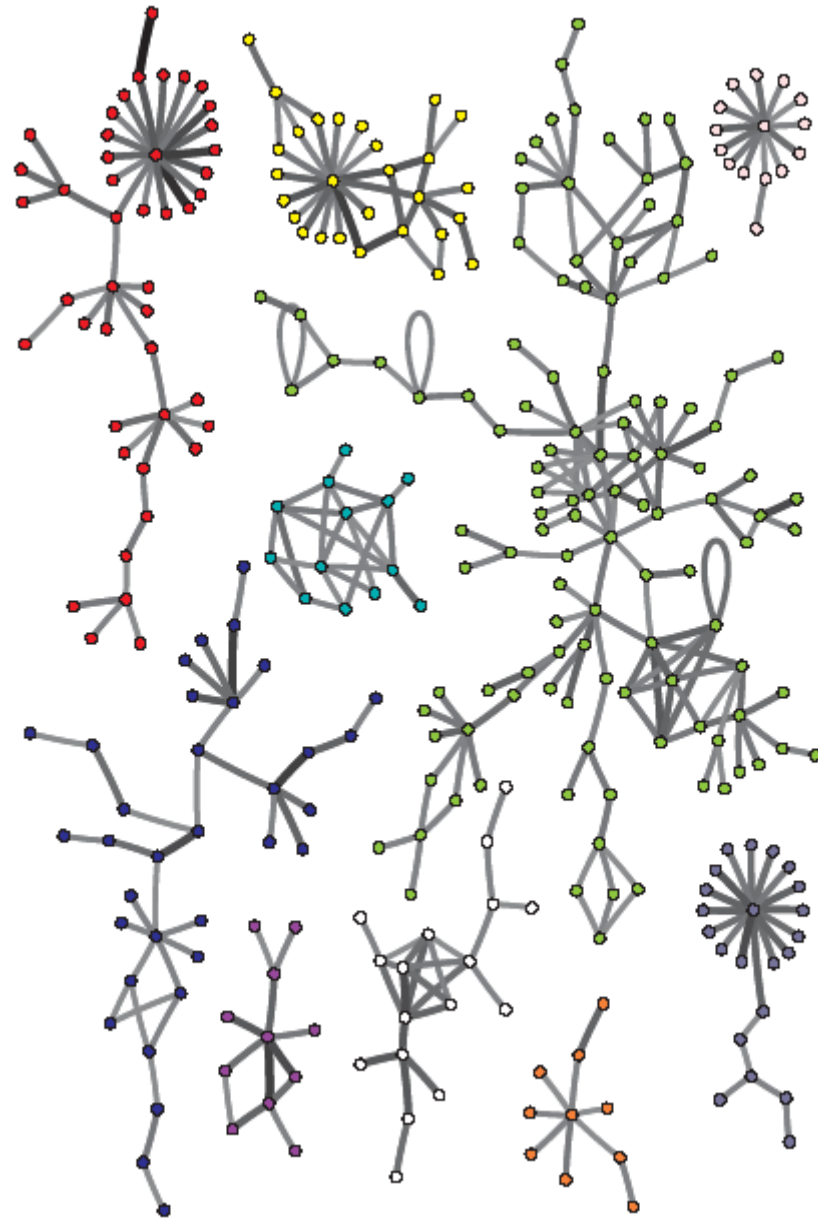
## Propagation of large concentration changes in reversible protein-binding networks

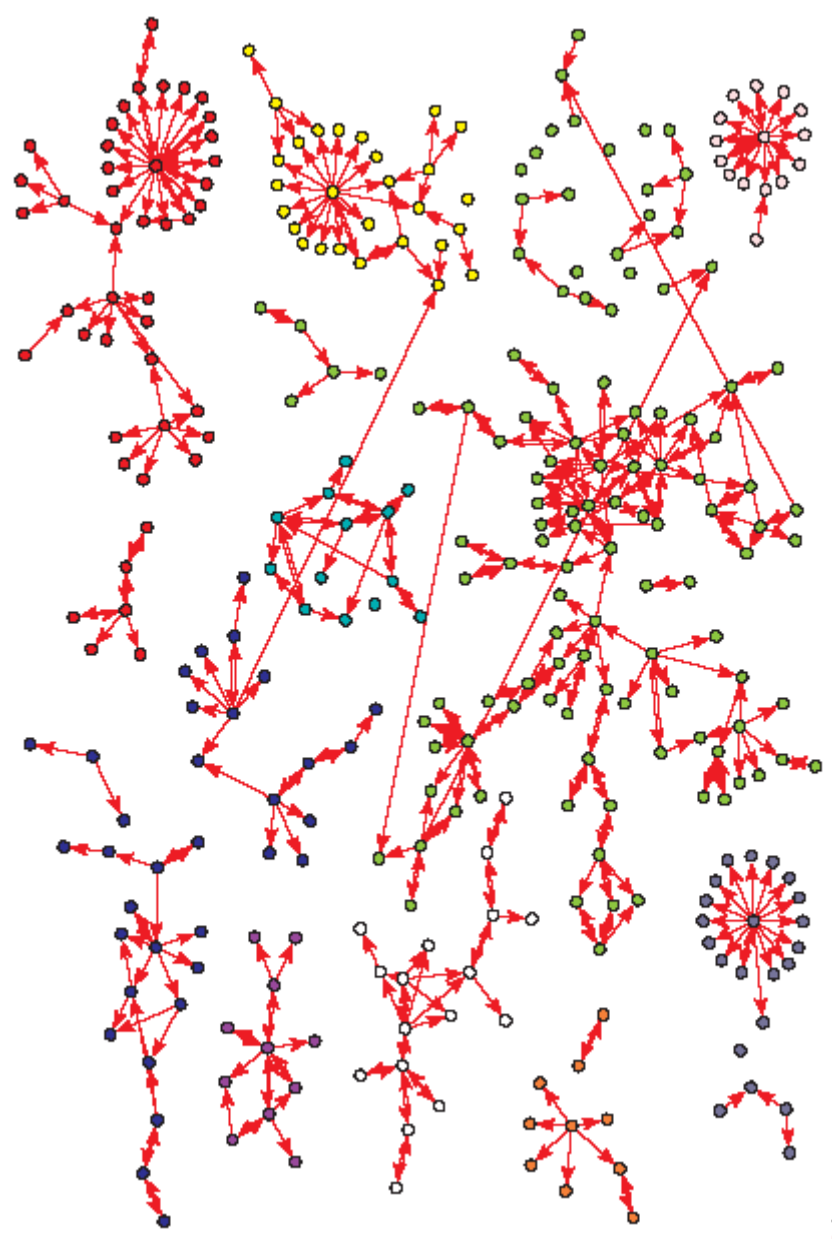
Sergei Maslov<sup>\*†</sup> and I. Ispolatov<sup>\*‡</sup>

<sup>\*</sup>Department of Condensed Matter Physics and Materials Science, Brookhaven National Laboratory, Upton, NY 11973; and <sup>†</sup>Ariadne Genomics, Inc., 9430 Key West Avenue, Suite 113, Rockville, MD 20850

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# Mapping to resistor network

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- Conductivities  $\sigma_{ij}$  – heterodimer concentrations  $D_{ij}$
- Losses to the ground  $\sigma_{iG}$  – free (unbound) concentrations  $F_i$
- Perturbations spread along linear chains loosely conducting to neighbors and ground
- Mapping is exact for bi-partite networks → odd-length loops dampen perturbations



# Collaborators and support

- Kim Sneppen (Center for Models of Life, NBI, Denmark)
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I have 1-2 postdoc positions to work on toolbox model.  
If interested talk to me

# Thank you!

